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Benchmark Example No. 8

Design of a Rectangular CS for Shear and Axial Force

VERiFiCATION
DCE-EN8 Design of a Rectangular CS for Shear and Axial Force

VERiFiCATION Manual, Service Pack 2024-4 Build 27

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The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.

Front Cover

6th Street Viaduct, Los Angeles Photo: Tobias Petschke

Overview

Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB
Input file(s):	rectangular_shear_axial.dat

1 Problem Description

The problem consists of a rectangular section, symmetrically reinforced for bending, as shown in Fig. 1. The cross-section is designed for a shear force V_{Ed} and a compressive force N_{Ed} and the required reinforcement is determined.

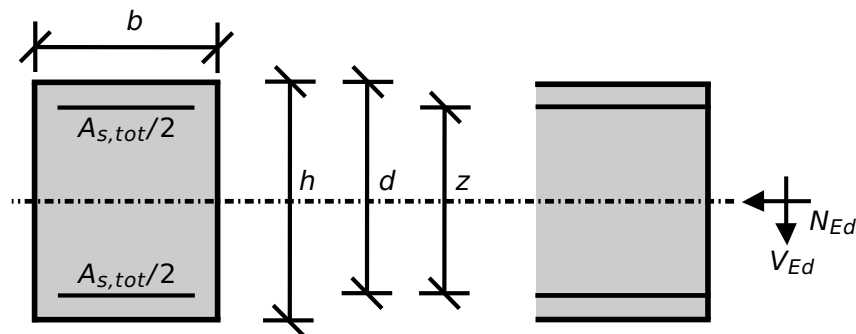


Figure 1: Problem Description

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to shear force and axial force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1] [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)
- Reinforcement (Section 9.2.2)

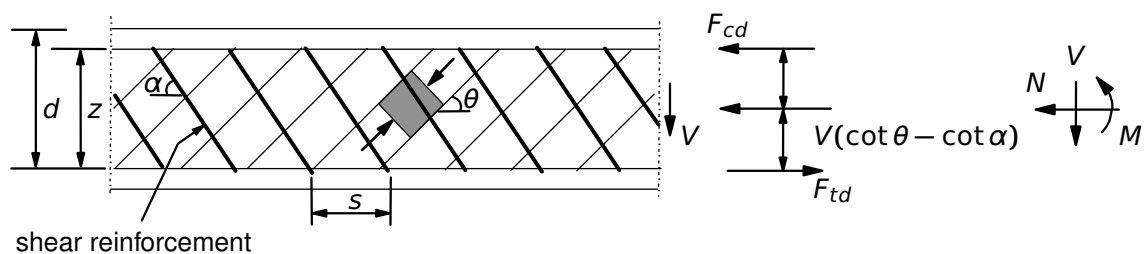


Figure 2: Shear Reinforced Members

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and defined in DIN EN 1992-1-1:2004 [1].

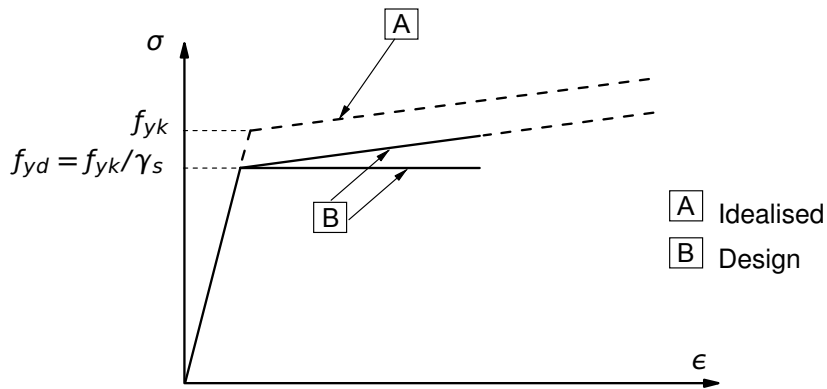


Figure 3: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The rectangular cross-section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], under a shear force of 343.25 kN and a compressive axial force of 500.0 kN. The reference calculation steps are presented below and the results are given in Table 2.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
C 30/37	$h = 50.0 \text{ cm}$	$V_{Ed} = 343.25 \text{ kN}$
B 500A	$b = 30 \text{ cm}$	$N_{Ed} = 500.0 \text{ kN}$
	$d = 45.0 \text{ cm}$	
	$A_{s,tot} = 38.67 \text{ cm}^2$	
	$c_{v,l} = 3.6 \text{ cm}$	

Table 2: Results

	SOF.	Ref.
$A_{sw} / s \text{ [cm}^2/\text{m]}$	11.27	11.27
$V_{Rd,c} \text{ [kN]}$	132.71	132.71
$\cot \theta$	1.82	1.82

4 Design Process¹

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:²

Material:

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

$f_{ck} = 30 \text{ MPa}$

$f_{cd} = a_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 30 / 1.5 = 17.0 \text{ MPa}$

$f_{yk} = 500 \text{ MPa}$

$f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 434.78 \text{ MPa}$

Design Load: $V_{Ed} = 343.25 \text{ kN}$

$N_{Ed} = -500.0 \text{ kN}$

$z = \max \{ d - c_{V,l} - 30 \text{ mm}; d - 2 c_{V,l} \}$

$z = \max \{ 384; 378 \} = 384 \text{ mm}$

$V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + 0.12 \cdot \sigma_{cp}] \cdot b_w \cdot d$

with a minimum of

$(v_{min} + 0.12 \cdot \sigma_{cp}) \cdot b_w \cdot d$

$C_{Rd,c} = 0.15 / \gamma_c = 0.1$

$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{450}} = 1.6667 < 2.0$

$\rho_1 = \frac{A_{s,tot}/2}{b_w d} = 0.01432 < 0.02$

$V_{Rd,c,min} = (v_{min} + 0.12 \cdot \sigma_{cp}) \cdot b_w \cdot d$

$v_{min} = (0.0525 / \gamma_c) \cdot k^{3/2} \cdot f_{ck}^{1/2} = 0.41249$

$V_{Rd,c,min} = 109.68 \text{ kN}$

$\sigma_{cp} = N_{Ed} / A_c < 0.2 \cdot f_{cd}$

$\sigma_{cp} = -500 \cdot 10^{-3} / 0.15 \cdot 10^6 = -3.3333 \text{ N/mm}^2 < 3.4$

$V_{Rd,c} = [0.1 \cdot 1.6667 \cdot (1.432 \cdot 30)^{1/3} + 0.112 \cdot 3.3333] \cdot 0.3 \cdot 0.45 = 132.71 \text{ kN}$

$V_{Ed} > V_{Rd,c} \rightarrow$ shear reinforcement is required

¹The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

²The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

Tab. 3.1: Strength for concrete

3.1.6: (1)P, Eq. (3.15): $a_{cc} = 0.85$ considering long term effects

3.2.2: (3)P: yield strength $f_{yk} = 500 \text{ MPa}$

3.2.7: (2), Fig. 3.8

(NDP) 6.2.3 (1): Inner lever arm z

(NDP) 6.2.2 (1): Eq. 6.2a: Design value for shear resistance $V_{Rd,c}$ for members not requiring design shear reinforcement

(NDP) 6.2.2 (1): Eq. 6.2b

(NDP) 6.2.2 (1): $C_{Rd,c} = 0.15 / \gamma_c$

(NDP) 6.2.2 (1): Eq. 6.3aDE: v_{min} for $d \leq 600 \text{ mm}$

(NDP) 6.2.2 (1): Eq. 6.2
 $\sigma_{cp} > 0$ for compression

(NDP) 6.2.3 (2): Eq. 6.7aDE

$$1.0 \leq \cot \theta \leq \frac{1.2 + 1.4 \sigma_{cd} / f_{cd}}{1 - V_{Rd,cc} / V_{Ed}} \leq 3.0$$

(NDP) 6.2.3 (2): Eq. 6.7bDE
 $c = 0.5$

$$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left(1 - 1.2 \frac{\sigma_{cd}}{f_{cd}}\right) \cdot b_w \cdot z$$

(NDP) 6.2.3 (2): $\sigma_{cd} = N_{Ed} / A_c$

$$\sigma_{cp} = N_{Ed} / A_c$$

$$\sigma_{cd} = -500 \cdot 10^{-3} / 0.15 \cdot 10^6 = -3.3333 \text{ N/mm}^2$$

6.7DE: $\sigma_{cd} > 0$ for compression

$$V_{Rd,cc} = 0.5 \cdot 0.48 \cdot 30^{1/3} \cdot \left(1 - 1.2 \frac{3.3333}{17.0}\right) \cdot 0.3 \cdot 0.384$$

$$V_{Rd,cc} = 65.6948 \text{ kN}$$

(NDP) 6.2.3 (2): The angle θ should be limited by Eq. 6.7DE

$$\cot \theta = \frac{1.2 + 1.4 \cdot 3.3333 / 17.0}{1 - 65.6948 / 343.25} = 1.823$$

6.2.3 (3): Eq. 6.8
 $f_{ywd} = f_{yk} / \gamma_s = 435 \text{ MPa}$

(NDP) 6.2.3 (3): Eq. 6.9
 Maximum shear force $V_{Rd,max}$ occurs for $\theta = 45^\circ$: $\cot \theta = \tan \theta = 1$
 (NDP) $v_1 = 0.75 \cdot v_2 = 0.75$, $v_2 = 1$ for $\leq C50/60$

$$A_{sw,requ} / s = V_{Ed} / (f_{ywd} \cdot z \cdot \cot \theta) = 11.27 \text{ cm}^2/\text{m}$$

$$V_{Rd,max} = b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot \theta + \tan \theta)$$

$$V_{Rd,max} = 0.3 \cdot 0.384 \cdot 0.75 \cdot 17 / (1 + 1) = 734.4 \text{ kN}$$

5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular beam cross-section under shear with compressive axial force. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

- [1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010.* CEN. 2010.
 - [2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau.* BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.
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