Benchmark Example No. 29

Design of restrained steel column

SOFiSTiK | 2024

VERiFiCATiON
DCE-EN29 Design of restrained steel column

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Front Cover

6th Street Viaduct, Los Angeles Photo: Tobias Petschke

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1993-1-1
Module(s):	BDK
Input file(s):	design_schneider_example_8-41.dat

1 Problem Description

The problem consists of a simply supported beam with a steel HEA 200 section which is restrained in the middle of the length. The column is subjecteded to compression and bending as shown in Fig. 1.

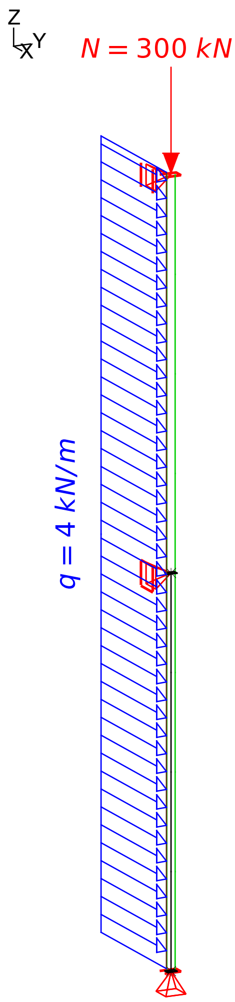

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the buckling resistance of steel members. It deals with the spatial behavior of the beam and the occurrence of lateral torsional buckling as a potential mode of failure. The content of this problem is covered by the following parts of DIN EN 1993-1-1:2005 [1]:

- Structural steel (Section 3.2)

- Classification of cross-sections (Section 5.5)
- Buckling resistance of members (Section 6.3)
- Method 2: Interaction factors k_{ij} for interaction formula in 6.3.3(4) (Annex B)

3 Model and Results

Table 1: Model Properties

Material Properties	Cross-Section Properties	Geometric Properties	Loading
$S\ 235$	$h = 190\ mm$	$H = 8.0\ m$	$q = 4.00\ kN/m$
$E = 210000\ N/mm^2$	$b = 200\ mm$		
$f_y = 235\ N/mm^2$	$c = 32\ mm$		
$\nu = 0.3$	$r = 18\ mm$		
$G = 81000\ N/mm^2$	$t_f = 10\ mm$		
$\gamma_{M0} = 1.0$	$t_w = 6.5\ mm$		
$\gamma_{M1} = 1.1$	$A = 5308\ mm^2$		
	$i_y = 82.8\ mm$		
	$i_z = 49.8\ mm$		
	$I_y = 3690\ cm^4$		
	$I_z = 1340\ cm^4$		
	$I_t = 21\ cm^4$		

Table 2: Results

	SOF.	Ref. [2]
$L_{cr,y}\ [m]$	8.00	8.00
$L_{cr,z}\ [m]$	4.00	4.00
$N_{pl,Rd}\ [kN]$	1265.4	1264.3
$M_{pl,y,Rd}\ [kNm]$	100.96	100.90
$\overline{\lambda}_y$	1.029	1.029
$\overline{\lambda}_z$	0.855	0.855
$N_{cr,z}\ [kN]$	1730.1	1736
$M_{cr}\ [kNm]$	245.47	220.9

Table 2: (continued)

	SOF.	Ref. [2]
$\overline{\lambda}_{LT}$	0.641	0.676
k_{yy}	1.292	1.304
k_{zy}	0.935	0.936

4 Design Process[1]

Design Loads:

$N_d = 300\ kN$

$q = 4.0\ kN/m$

$M_{y,d} = q \cdot L^2/8 = 4.0 \cdot 8^2/8 = 32.0\ kNm$

$M_{z,d} = 0$

Buckling lengths:

$L_{cr,y} = 8.00\ m$

$L_{cr,z} = 4.00\ m$

$\xi = 1.35$

Characteristic values:

$N_{Rk} = N_{pl,Rd} = 1264.3\ kN$

$M_{y,Rk} = M_{pl,y,Rd} = 100.9\ kNm$

I_z - second moment of area

$I_z = 1340\ cm^4$

I_w - warping resistance

$I_w = 108000\ cm^6$

I_t - torsional moment of inertia

$I_t = 21.0\ cm^4$

$i_y = 8.28\ cm$

$i_z = 4.98\ cm$

$z_p = -9.5\ cm$

Buckling around the y-y axis:

$\overline{\lambda}_y = 800/(8.28 \cdot 93.9) = 1.029 \rightarrow \chi = 0.58$ (Curve b)

$\overline{\lambda}_z = 400/(4.98 \cdot 93.9) = 0.855 \rightarrow \chi = 0.63$ (Curve c)

Critical loading:

N_{cr} - elastic critical force for the relevant buckling mode based on the gross cross sectional properties

$N_{cr,z} = \pi^2 \cdot 21000 \cdot 1340/400^2 = 1736\ kN$

$c^2 = (108000 + 0.039 \cdot 400^2 \cdot 21.0)/1340 = 178.4\ cm^2$

M_{cr} - elastic critical moment for lateral-torsional buckling

$M_{cr} = 1.35 \cdot 1736 \cdot \left[\sqrt{178.4 + 0.25 \cdot 9.5^2} + 0.5 \cdot (-9.5)\right] \cdot 10^2$

$M_{cr} = 220.9\ kNm$

[1]The sections mentioned in the margins refer to DIN EN 1993-1-1:2005 [1] unless otherwise specified.

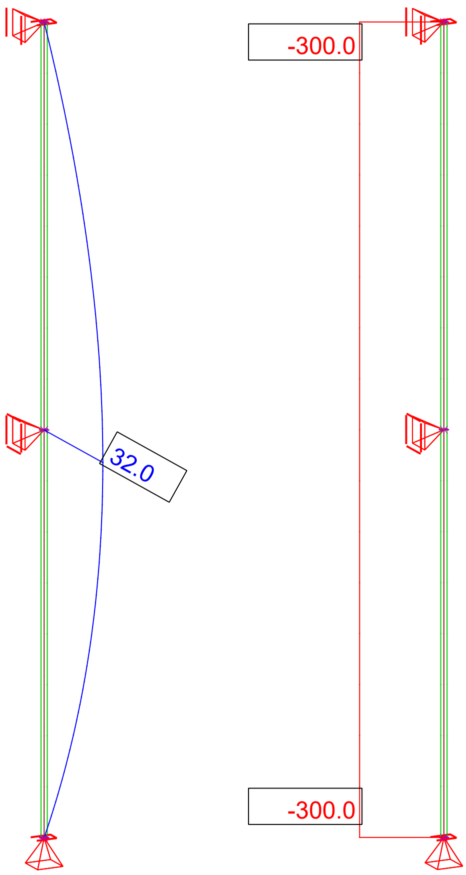

Figure 2: Internal forces M_y and N

$\overline{\lambda}_{LT} = \sqrt{100.9/220.9} = 0.676$

$\overline{\lambda}_{LT}$ - non dimensional slenderness for lateral torsional buckling

$h/b = 190/200 = 0.95 < 2.0 \rightarrow$ Class b

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta \cdot \overline{\lambda}_{LT}^2}}$$

$$\Phi_{LT} = 0.5 \cdot \left[1 + \alpha_{LT} \cdot (\overline{\lambda}_{LT} - \overline{\lambda}_{LT,0}) + \beta \cdot \overline{\lambda}_{LT}^2\right]$$

$\chi_{LT} = 0.88 < 2.188 = 1/0.676^2$

χ_{LT} - reduction factor

$$M_{b,Rd} = M_{pl,y,Rd} \cdot \frac{\chi_{LT}}{f} \cdot \frac{1}{\gamma_{M1}}$$

$$f = 1 - 0.5 \cdot (1 - k_c) \cdot \left[1 - 2.0 \cdot (\overline{\lambda}_{LT} - 0.8^2)\right]$$

f - the value f may be defined in National Annex, see EC 3, §6.3.2.3(2)

$$f = 1 - 0.5 \cdot (1 - 0.94) \cdot \left[1 - 2.0 \cdot (0.676 - 0.8^2)\right]$$

$f = 0.972$

$$M_{b,Rd} = 100.9 \cdot \frac{0.88}{0.972} \cdot \frac{1}{1.1}$$

$M_{b,Rd} = 83.045\ kNm$

$m_{b,Rd}$ - design buckling resistance moment - EC 3, §6.3.2.1, Eq. 6.55

Equivalent uniform moment factors:

$L_{cr,y}$ - buckling length

for $L_{cr,y} = 8.00\ m$:

$$\alpha_h = M_h/M_s = 0.00$$

$$\psi = 1.00$$

$$c_{my} = 0.95 + 0.05 \cdot 0.00 = 0.95$$

$L_{cr,z}$ - buckling length

for $L_{cr,z} = 4.00\ m$:

$$\alpha_h = M_h/M_s = 0.75$$

$$\psi = 0.00$$

$$c_{mLT} = 0.2 + 0.8 \cdot 0.75 = 0.80$$

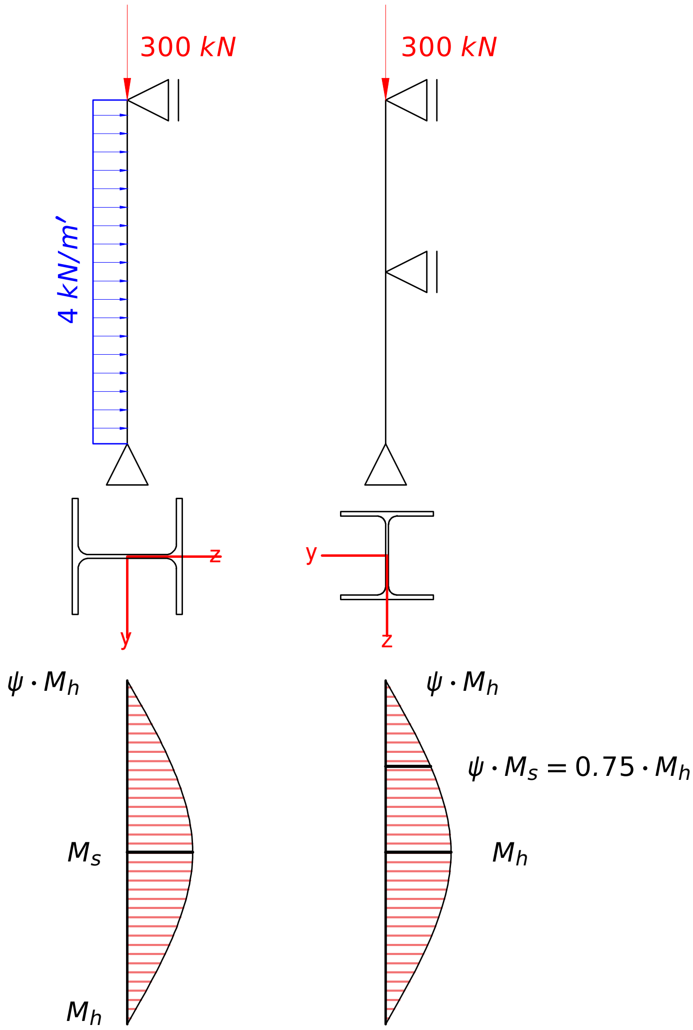

Figure 3: Calculating the equivalent uniform moment factors

k_{yy}, k_{zy} - Annex B: Method 2, interaction factors for interaction formula in §6.3.3(4)

Interaction factors:

$$
\begin{aligned}
k_{yy} \quad &= 0.95 \cdot \left(1 + 1.029 - 0.2) \cdot 300 \cdot \frac{1.1}{0.580 \cdot 1264.3}\right) \\
&= 1.304 \\
&\leq 0.95 \cdot \left(1 + 0.8 \cdot 300 \cdot \frac{1.1}{0.580 \cdot 1264.3}\right) = 1.292 \\
&\leq 1.80
\end{aligned}
$$

$$
\begin{aligned}
k_{zy} \quad &= \left[1 - \frac{0.1 \cdot 0.855}{0.8 - 0.25} \cdot \frac{300 \cdot 1.1}{0.63 \cdot 1264.3}\right] = 0.936 \\
&\geq \left[1 - \frac{0.1}{0.8 - 0.25} \cdot \frac{300 \cdot 1.1}{0.63 \cdot 1264.3}\right] = 0.925 \\
&\geq 0.3 \cdot 300 \cdot \frac{1.1}{0.63 \cdot 1264.3} = 0.138
\end{aligned}
$$

Lateral Torsional Buckling check:

$$
\frac{300}{0.58 \cdot 1264.3/1.1} + 1.292 \cdot \frac{32.0 + 0.0}{0.88 \cdot 100.9/1.1} + 0.0 = 0.96 \leq 1.0
$$

Lateral Torsional Buckling, §6.3.3, Eq. 6.61 and 6.62

$$
\frac{300}{0.63 \cdot 1264.3/1.1} + 0.936 \cdot \frac{32.0 + 0.0}{0.88 \cdot 100.9/1.1} + 0.0 = 0.79 \leq 1.0
$$

5 Conclusion

This example shows the check for lateral torsional buckling of steel members. The small deviations that occur in some results come from the fact that there are some small differences in the sectional values and elastic critical loadings (M_{cr}, N_{cr}). Therefore, these deviations are of no interest for the specific verification process. In conclusion, it has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *DIN EN 1993-1-1:2005 Eurocode 3: Design of steel structures, Part 1-1: General rules and rules for buildings - Deutsche Fassung EN 1993-1-1:2005 + AC:2009*. CEN. 2010.

[2] Schneider. *Bautabellen für Ingenieure*. 21th. Bundesanzeiger Verlag, 2014.