Benchmark Example No. 19

Fatigue of a Rectangular Reinforced Concrete CS

SOFiSTiK | 2024

VERiFiCATiON
DCE-EN19 Fatigue of a Rectangular Reinforced Concrete CS

VERiFiCATiON Manual, Service Pack 2024-4 Build 27

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Front Cover

6th Street Viaduct, Los Angeles   Photo: Tobias Petschke

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB
Input file(s):	fatigue.dat

1 Problem Description

The problem consists of a simply supported box girder beam of reinforced concrete, as shown in Fig. 1. The structure's resistance to fatigue shall be verified.

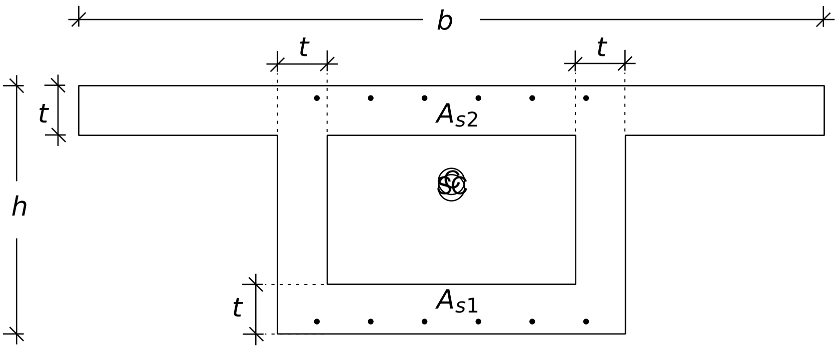

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the verification to fatigue. The content of this problem is covered by the following parts of DIN EN 1992-1-1/NA [1] [2]:

- Verification conditions (Section 6.8.1)
- Internal forces and stresses for fatigue verification (Section 6.8.2)
- Combination of actions (Section 6.8.3)
- Verification procedure for reinforcing and prestressing steel (Section 6.8.4)
- Verification using damage equivalent stress range(Section 6.8.5)
- Verification of concrete under compression or shear (Section 6.8.7)

3 Model and Results

The properties of the simply supported beam of reinforced concrete with a box cross-section are defined in Table 1. The beam is loaded with three combinations of load cases with calculatoric forces and moments, as presented in Table 1. A verification of its resistance to fatigue is performed at $x = 5\ m$ with respect to DIN EN 1992-1-1/NA [1] [2]. The results are given in Table 2

Table 1: Model Properties

Material	Geometry	Loading (at $x = 5\ m$)
$C\ 35/45$	$h = 200.0\ cm$	LC 911:
$S\ 500$	$b = 600.0\ cm$	$V_z = 610\ kN$, $M_y = 4575\ kNm$, $M_t = -0.19\ kNm$
	$t = 400.0\ cm$	LC 912:
	$L = 20.0\ m$	$V_z = 660\ kN$, $M_y = 4950\ kNm$, $M_t = -50.20\ kNm$
	$A_{s1} = 60\ cm^2$	LC 913:
	$A_{s2} = 60\ cm^2$	$V_z = 710\ kN$, $M_y = 5325\ kNm$, $M_t = 99.78\ kNm$

Table 2: Results

Result	SOF (FEM).	Ref.
$\Delta_{\sigma_{s,equ}}(N^*)\ [MPa]$	74.04	76.98
$f_{cd,fat}\ [MPa]$	17.06	17.0567
$\sigma_{cd,max,equ'_{TOP}'}\ [MPa]$	≤ 14.33	≤ 14.33
$\sigma_{cd,max,equ_{shear\ cut}}\ [MPa]$	≤ 10.39	≤ 10.35
$\frac{\Delta_{\sigma_{Rsk}}(N^*)}{\gamma_{s,fat}}\ [MPa]$	152.17	152.2

4 Design Process[1]

Design with respect to DIN EN 1992-1-1/NA [1] [2]:[2]

STEP 1: Material

Concrete: $C\ 35/45$

3.1: Concrete

$f_{ck} = 35\ N/mm^2$

Tab. 3.1: Strength for concrete

$\gamma_c = 1.50$

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 0.85 \cdot 35/1.5 = 19.83\ MPa$

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

STEP 2: Cross-section

$1/W_{V_z} = 0.8177 \quad 1/m^2$

$1/W_{V_y} = 0.371 \quad 1/m^2$

$1/W_T = 0.3448 \quad 1/m^3$

$1/W_{V_i}$: Shear force resistance, calculated by using BEM

$1/W_T$: Torsional resistance, calculated by using BEM

Minimun reinforcements:

$A_{s1} = A_{s2} = 6 \cdot 10 = 60\ cm^2$

$A_{sl} = 8.22\ cm^2/m$

A_{si}: Longitudinal

A_{sl}: Shear links

STEP 3: Load Actions:

Permanent: Loadcase 1

Variable: Loadcase 2, 3

For the determination of the combination calculatoric forces and moments the following superposition types are chosen:

- Quasi permanent combination for serviceability - MAXP
- Frequent combination for serviceability - MAXF

6.8.3: (2)P: Fatigue
The basic combination of the non-cyclic load is similar to the definition of the frequent combination for serviceability:
$\sum_{j\geq1} G_{k,j}\ {''}+{''}\ P\ {''}+{''}\ \psi_{1,1} Q_{k,1}\ {''}+{''}\ {''}+{''}\ \sum_{i>1} \psi_{2,i} Q_{k,i}$

The following combination of actions scenario is investigated for serviceability:

- LC 911 G
  MAXP + MY : 1.00 * G
- LC 912 G+2
  MAXF + MY : 1.00 * G + ψ_1 * LC 2
- LC 913 G+3
  MAXF + MY : 1.00 * G + ψ_1 * LC 3

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [2] 3.3.6: Fig. 3.10

[2] The sections mentioned in the margins refer to DIN EN 1992-1-1/NA [1], [2], unless otherwise specified.

Combination calculatoric forces and moments at $x = 5.0\ m$:

LC	V_y $[kN]$	V_z $[kN]$	M_y $[kNm]$	M_t $[kNm]$
911	0	610	4575	−0.189
912	0	660	4950	−50.20
913	0	710	5325	99.78

STEP 4: Calculation of stresses at $x = 5.0\ m$:

The resistance of structures to fatigue shall be verified in special cases.

6.8.1 (1)P: Verification conditions

This verification shall be performed separately from concrete and steel.

The following calculation corresponds to LC 911.

τ_Q: shear stresses resulting from shear force

$\tau_Q = 1/W_{V_y} \cdot Q_y + 1/W_{V_z} \cdot Q_z$

where Q_y and Q_z are calculated through a proportionate factor f_V, depending on the lever arm of internal forces and the elastic part of V_y and V_z.

The proportionate factor f_V is obtained from the internal lever in cracked condition to the un-cracked condition.

$$V_I = \sqrt{V_y^2 + V_z^2} = \sqrt{0^2 + 610^2} = 610\ kN$$

$$V_{II} = \sqrt{\left(\frac{V_y}{z_{y,II}}\right)^2 + \left(\frac{V_z}{z_{z,II}}\right)^2}$$

$$V_{II} = \sqrt{\left(\frac{0}{3.369}\right)^2 + \left(\frac{610}{1.528}\right)^2} = 399.21\ kN$$

$$f_V = min\left(1, \frac{\left|\frac{V_I}{z_0}\right|}{V_{II}}\right) = min\left(1, \frac{\left|\frac{610}{1.782}\right|}{399.21}\right) = 0.8576$$

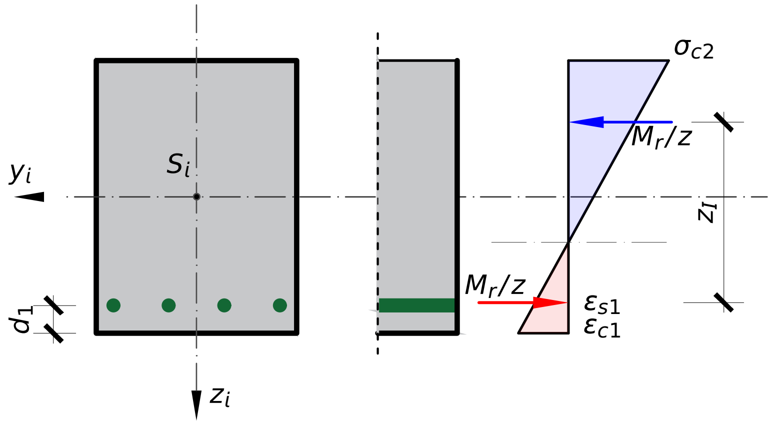

Figure 2: Stress distribution in un-cracked state - z_I

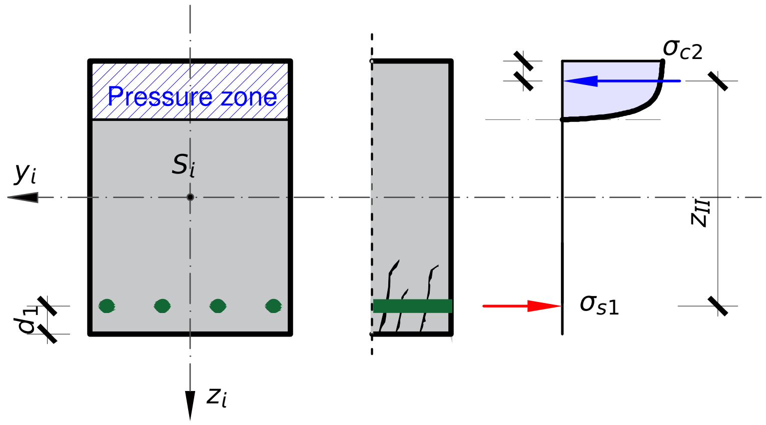

Figure 3: Stress distribution in cracked state - z_{II}

$$Q_y = f_V \cdot V_y = 0.8576 \cdot 0.0 = 0.0\ kN$$

$$Q_z = f_V \cdot V_z = 0.8576 \cdot 610 = 523.149\ kN$$

$$\tau_Q = 0.371 \cdot 0.0 + 0.8177 \cdot 523.149 = 427.770 \cdot 10^{-3}\ MPa$$

$$\tau_T = -1/W_T \cdot M_t = -0.344484 \cdot -0.189 = 0.065 \cdot 10^{-3}\ MPa$$

τ_T: shear stresses resulting from torsion

$$\tau = \tau_Q + \tau_T = 427.770 \cdot 10^{-3} + 0.065 \cdot 10^{-3} = 427.835 \cdot 10^{-3}\ MPa$$

$$\tau_{II} = (\tau_Q + \tau_T) \cdot (1.0 + \cot^2\theta)$$

τ_{II}, σ_{II}: principal stresses

$$\sigma_{II} = \frac{\tau_{II}}{\cot\theta + \cot\alpha}$$

θ: angle of compression struts
α: angle of shear reinforcement
$\alpha = 90^o \Rightarrow \sin\alpha = 1.0, \cot\alpha = 0.0$

A rather nasty problem is the evaluation of the shear. The DIN design code allows a simple solution based on a corrected value for the inclination of the compressive struts:

$$\tan\theta_{fat} = \sqrt{\tan\theta}$$

6.8.2(3): In the design of shear reinforcement the inclination of the compressive struts θ_{fat} may be calculated by Eq. 6.65

6.8.2(3): Eq. 6.65: $\tan\theta_{fat}$

Unfortunately it is nearly impossible to keep this value from the shear design for all individual shear cuts or transform it to different load combinations and reinforcement distributions for the fatigue stress check. **AQB** uses instead a fixed value of 4/7 for the tangents. The user may overwrite this value however with any desired value.

$$\tan\theta = 4/7 \Rightarrow \cot\theta = 7/4 = 1.75$$

$$\tan\theta_{fat} = \sqrt{4/7} = 0.756$$

$$\cot\theta_{fat} = \sqrt{7/4} = 1.3229$$

$$\tau_{II} = (427.770 \cdot 10^{-3} + 0.065 \cdot 10^{-3}) \cdot (1.0 + 1.75^2)$$

$$\tau_{II} = 1740.048 \cdot 10^{-3}\ MPa$$

$$\sigma_{II} = \frac{1740.048 \cdot 10^{-3}}{1.75 + 0.0} = 994.313 \cdot 10^{-3}\ MPa$$

σ_{sl}: steel stresses

$$\sigma_{sl} = \frac{f_Q \cdot \tau_Q}{(\cot\theta_{fat} + \cot\alpha) \cdot \sin\alpha} + \frac{f_T \cdot \tau_T}{\cot\theta_{fat}}$$

$A_{sl/cut} = A_{sl}\ /\ 2 = 4.11\ cm^2/m$

$$f_T = \frac{B_0 \cdot f_r}{A_{sl/cut}}, \quad \text{and} \quad f_Q = \frac{B_b \cdot f_r}{A_{sl/cut}}$$

where f_T and f_Q are factors expressing the shear links reinforcement ratios. They are depending on f_r, a factor for total reinforcement, B_0, the width of the cut and B_b, the total width of the cut. Since in this case it is a box cross-section and taking into account the position of the cut, we get that $B_0 = B_b = 0.4\ m$.

The factor f_r has only two possible values $f_r = 1.0$ or $f_r = 2.0$. It depends on the cross-section and the shear cut. If $B_{max} < B_0$ then $f_r = 2.0$.

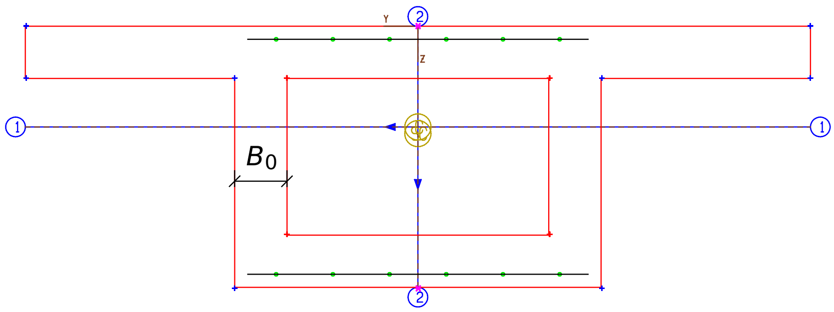

Figure 4: Cross-section Overview

$$f_T = \frac{0.4 \cdot 1.0}{4.11 \cdot 10^{-4}} = 973.532 \quad \text{and} \quad f_Q = \frac{0.4 \cdot 1.0}{4.11 \cdot 10^{-4}} = 973.532$$

$$\sigma_{sl} = \frac{973.532 \cdot 427.770 \cdot 10^{-3}}{(1.3229 + 0.0) \cdot 1.0} + \frac{973.532 \cdot 0.065 \cdot 10^{-3}}{1.3229}$$

$$\sigma_{sl} = 314.882\ MPa$$

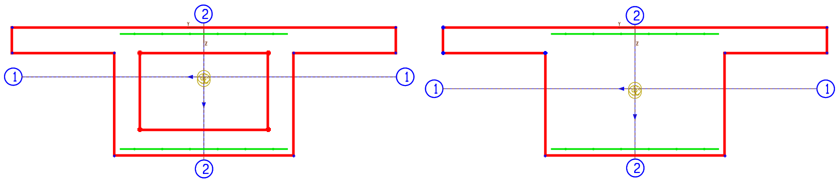

Figure 5: Factor of total reinforcement, $f_r = 1.0$ (left), $f_r = 2.0$ (right)

Accordingly, we calculate the stresses for the rest of the loadcases. For each loadcase the stresses are calculated for two cases, for τ_T and for $-\tau_T$, in order to determine the most unfavorable case. The results are presented in Table 4

Table 4: Calculation of Stresses by using BEM

LC	Q_z [kN]	τ_Q [MPa]	$\tau_T \cdot 10^{-3}$ [MPa]	τ [MPa]	τ_{II} [MPa]	σ_{II} [MPa]	σ_{sl} [MPa]
911	523.14	0.428	0.065	0.427	1.740	0.993	314.88
			-0.065	0.427	1.739	0.993	314.78
912	566.03	0.463	17.31	0.480	1.952	1.115	353.38
			-17.31	0.4455	1.8120	1.0346	327.90
913	608.911	0.498	-34.41	0.4635	1.8851	1.0764	341.12
			34.41	0.5323	2.1649	1.2362	391.7

From the table, the minimum and maximum value of the steel stress is determined:

- Max. $\sigma_{sl} = 391.77\ MPa$
- Min. $\sigma_{sl} = 314.79\ MPa$

As the exact fatigue stress check is not available, the simplified methods according to DIN EN 1992-1-1/NA (Sect. 6.8, Fatigue) are selected via the coefficients λ_s, λ_t, λ_l, λ_c.

λ_l: Coeff. equiv. stress range shear links, here input as 1.0

The admissible sways of the damage equivalent stress range for the shear links are obtained, as follows:

$$\Delta_{\sigma_{s,equ}}(N^*) = \lambda_l \cdot (\sigma_{sl,max} - \sigma_{sl,min}) = 1.0 \cdot (391.77 - 314.79)$$

$$\Delta_{\sigma_{s,equ}}(N^*) = 76.98\ MPa$$

6.8.2 (2): Eq. 6.64: η factor for effect of different bond behaviour $\eta = \frac{A_s + A_p}{A_s + A_p\sqrt{\xi \cdot \phi_s/\phi_p}}$, since $A_p = 0$ (no prestress) $\Rightarrow \eta = 1.0$, thus no increase of calculated stress range in the reinforcing steel

For reinforcing steel adequate fatigue resistance should be assumed, if the following is satisfied:

$$\gamma_{F,fat} \cdot \Delta_{\sigma_{s,equ}}(N^*) \leq \frac{\Delta_{\sigma_{Rsk}}(N^*)}{\gamma_{s,fat}}$$

$$1.0 \cdot 76.98 \leq \frac{175}{1.15}$$

6.8.5 (3): Eq. 6.71: Verification using damage equivalent stress range

(NDP) 6.8.4 (6): Table 6.3DE: Parameters for fatigue strength curves for reinforcing steel $\Delta_{\sigma_{Rsk}}(N^*) = 175$ for straight/bent bars and $N^* = 10^6$ cycles

(NDP) 2.4.2.3 (1): Partial factor for fatigue loads $\gamma_{F,fat} = 1.0$

(NDP) 2.4.2.4 (1): Partial factors for materials $\gamma_{s,fat} = 1.15$

$\Delta_{\sigma_{s,equ}}(N^*) = 76.98 \leq 152.2\ MPa$

If a coefficient $\lambda_l = 2.0$ is input for the shear links, resulting in a stress range of $\Delta_{\sigma_{s,equ}}(N^*) = 2 \cdot 76.98 = 153.97\ MPa$, a star $(^*)$ will be printed in the output next to the shear link stress range, denoting that the limit value of $152.2\ MPa$ has been exceeded.

(NDP) 6.8.7 (1): Eq. 6.76: $f_{cd,fat}$
$k_1 = 1.0$
3.1.2 (6): Eq. 3.2: $\beta_{cc}(t_0)$
$\beta_{cc}(t_0) = e^{s\cdot(1-\sqrt{28/t}} = 1.0$

The design fatigue strength of concrete is determined by:

$$f_{cd,fat} = k_1 \cdot \beta_{cc}(t_0) \cdot f_{cd} \cdot \left(1 - \frac{f_{ck}}{250}\right)$$

$$f_{cd,fat} = 1.0 \cdot 1.0 \cdot 19.83 \cdot \left(1 - \frac{35}{250}\right) = 17.0567\ MPa$$

(NDP) 6.8.7 (3): Verification of concrete under compression or shear

In the case of the compression struts of members subjected to shear, the concrete strength $f_{cd,fat}$ should be reduced by the strength reduction factor ν_1 according to 6.2.3(3).

(NDP) 6.2.3 (3): ν_1, ν_2

$$\nu_2 = (1.1 - \frac{f_{ck}}{500}) \leq 1.0$$

$$\nu_2 = (1.1 - \frac{35}{500}) = 1.03 \rightarrow \nu_2 = 1.0$$

$$\nu_1 = 0.75 \cdot \nu_2$$

$$\nu_1 = 0.75 \cdot 1.0 = 0.75$$

$$\Rightarrow f_{cd,fat,red} = 0.75 \cdot 17.0567 = 12.7925\ MPa$$

A satisfactory fatigue resistance may be assumed, if the following condition is fulfilled:

6.8.7 (1): Eq. 6.72 - 6.75

$$E_{cd,max,equ} + 0.43 \cdot \sqrt{1 - R_{equ}} \leq 1$$

$$\frac{\sigma_{cd,max,equ}}{f_{cd,fat,red}} + 0.43 \cdot \sqrt{1 - \frac{\sigma_{cd,min,equ}}{\sigma_{cd,max,equ}}} \leq 1$$

$$\sigma_{cd,max,equ} \leq f_{cd,fat,red} \cdot \left(1.0 - 0.43 \cdot \sqrt{1 - \frac{\sigma_{cd,min,equ}}{\sigma_{cd,max,equ}}}\right)$$

$$\sigma_{cd,max,equ} \leq 12.7925 \cdot \left(1.0 - 0.43 \cdot \sqrt{1 - \frac{0.9933}{1.2362}}\right)$$

$$\sigma_{cd,max,equ} = 1.2362 \leq 10.35\ MPa$$

Accordingly the above verification is done for the minimum and maximum nonlinear stresses of concrete, as calculated from **AQB** Fig. 6, at the defined $'TOP'$ point of the cross-section.

$$\sigma_{cd,max,equ} \leq f_{cd,fat} \cdot \left(1.0 - 0.43 \cdot \sqrt{1 - \frac{\sigma_{cd,min,equ}}{\sigma_{cd,max,equ}}}\right)$$

$$\sigma_{cd,max,equ} \leq 17.0567 \cdot \left(1.0 - 0.43 \cdot \sqrt{1 - \frac{5.69}{6.60}}\right)$$

$\sigma_{cd,max,equ} = 6.60 \leq 14.33\ MPa$

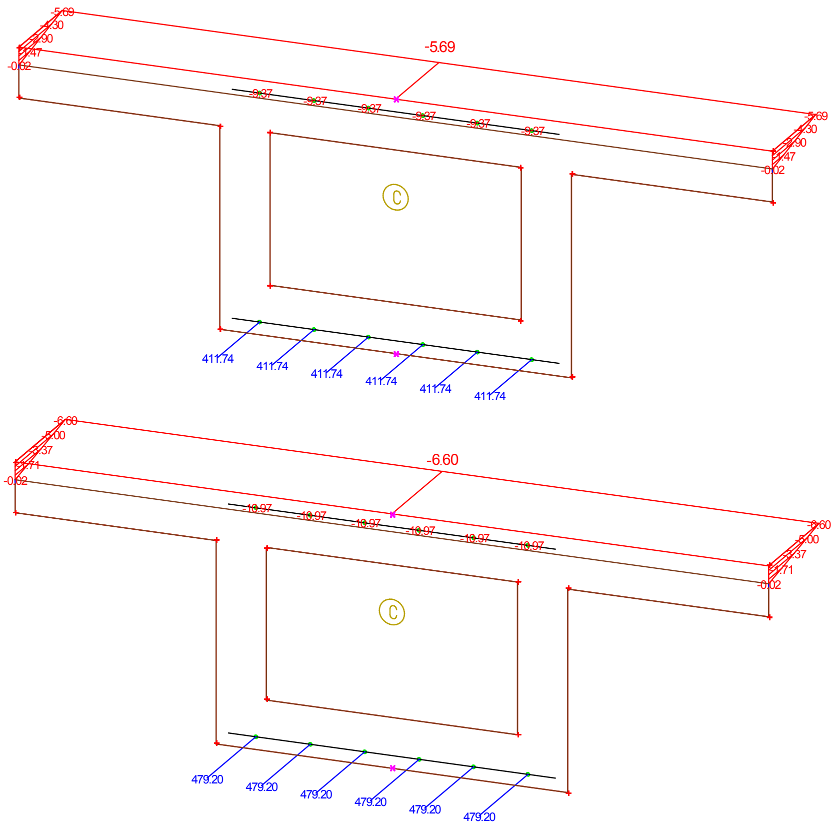

Figure 6: Min/Max. Nonlinear Stresses of Concrete at "TOP" Point (BEM)

Stress limitation:

$$\sigma_{max,t} = k_3 \cdot f_{yk} = 0.80 \cdot 500\ MPa = 400\ MPa$$ 7.2 (5)

5 Conclusion

This example shows the verification of a reinforced concrete beam to fatigue. It has been shown that **AQB** follows the fatigue verification procedure, as proposed in DIN EN 1992-1-1/NA [1] [2]. The insignificant deviation arises from the fact that the benchmark (reference) results have been calculated by using the BEM analysis. By introducing the FEM analysis, AQUA calculates now the $1/W_{Vz}$, $1/W_{Vy}$ and $1/W_T$ values more accurate.

6 Literature

[1] *DIN EN 1992-1-1/NA:2013-04: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - Nationaler Anhang Deutschland, Ersatz für DIN EN 1992-1-1/NA:2011-01 und DIN EN 1992-1-1/NA Berichtigung 1:2012-06*. DIN. 2013.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.