Benchmark Example No. 17

Stress Calculation at a Rectangular Prestressed Concrete CS

SOFiSTiK | 2024

VERiFiCATiON
DCE-EN17 Stress Calculation at a Rectangular Prestressed Concrete CS

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Front Cover

6th Street Viaduct, Los Angeles   Photo: Tobias Petschke

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB, TENDON
Input file(s):	stress_prestress.dat

1 Problem Description

The problem consists of a rectangular cross-section of prestressed concrete, as shown in Fig. 1. The stresses developed at the section due to prestress and bending are verified.

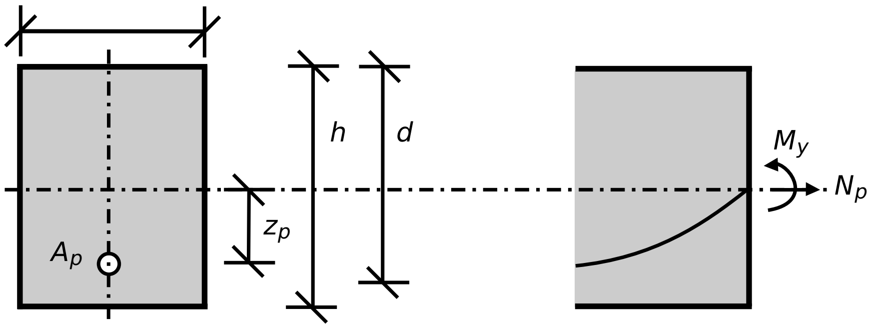


Figure 1: Problem Description

2 Reference Solution

This example is concerned with the design of prestressed concrete cs, subject to bending and prestress force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Stress-strain curves for concrete and prestressing steel (Section 3.1.7, 3.3.6)
- Verification by the partial factor method - Design values (Section 2.4.2)
- Prestressing force (Section 5.10.2, 5.10.3)

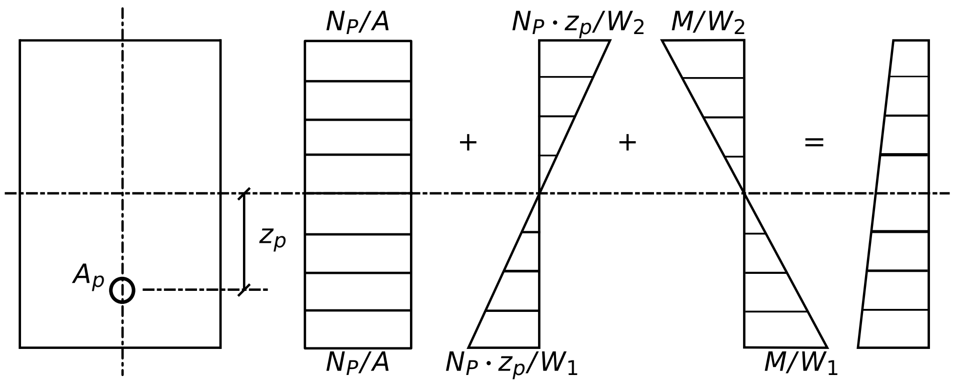


Figure 2: Stress Distribution in Prestress Concrete Cross-section

In rectangular cs, which are prestressed and loaded, stress conditions are developed, as shown in Fig. 2, where the different contributions of the loadings can be seen. The design stress-strain diagrams for prestressing steel is presented in Fig. 3, as defined in DIN EN 1992-1-1:2004 [1] (Section 3.3.6).

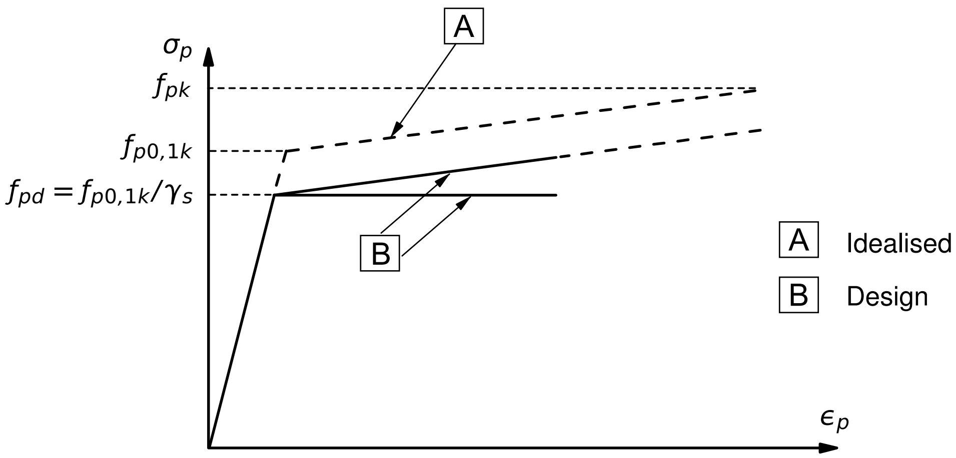


Figure 3: Idealised and Design Stress-Strain Diagram for Prestressing Steel

3 Model and Results

The simply supported beam of Fig. 4, consists of a rectangular cross-section with properties as defined in Table 1 and is prestressed and loaded with its own weight. A verification of the stresses is performed in the middle of the span with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2]. The geometry of the tendon can be visualised in Fig. 5. The calculation steps [3] are presented below and the results are given in Table 2.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading (at $x = 10$ m)
C 35/45	$h = 100.0$ cm	$M_g = 1250$ kNm
Y 1770	$b = 100.0$ cm	$N_p = -3651.1$ kN
	$d = 95.0$ cm	
	$L = 20.0$ m	
	$A_p = 28.5$ cm^2	

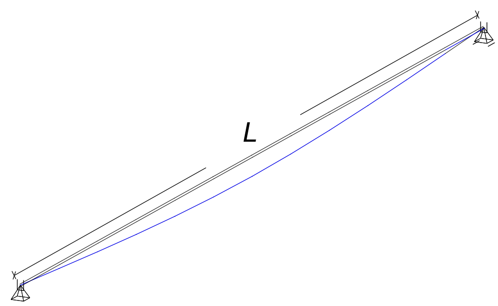


Figure 4: Simply Supported Beam

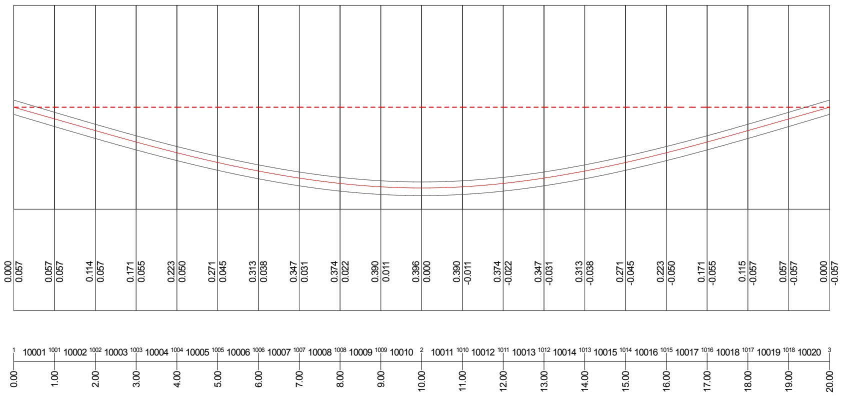


Figure 5: Tendon Geometry

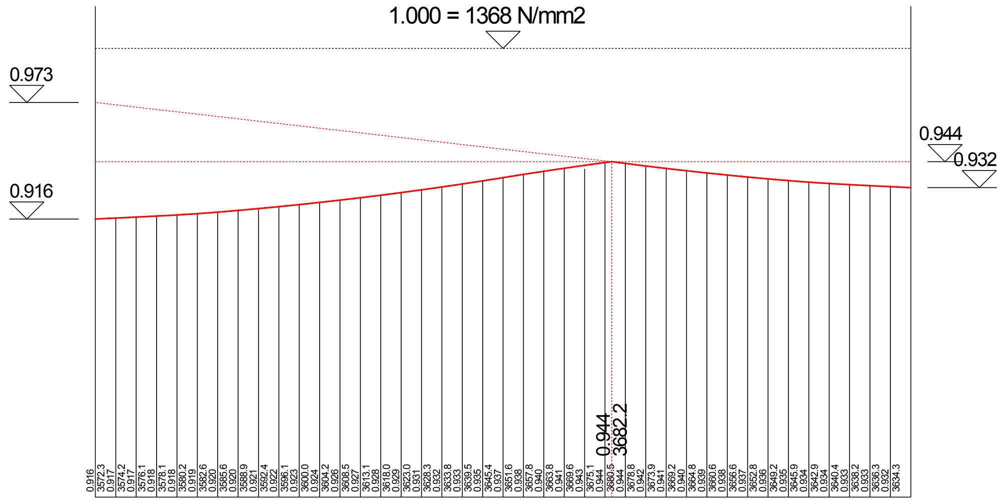


Figure 6: Prestress Forces and Stresses

Table 2: Results

Case	CS	Result	SOF.	Ref.
I	0	$\sigma_{c,b}$ $[MPa]$	-12.47	-12.47
		M_y $[kNm]$	-1435.88	-1435.91
II	0		-4.82	-4.82
		M_y $[kNm]$	-185.88	-185.91
III	1		-11.76	-11.76
		M_y $[kNm]$	-1406.08	-1406.11
IV	1		-4.51	-4.51
		M_y $[kNm]$	-156.08	-156.11

4 Design Process[1]

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

Material:

3.1: Concrete

Concrete: $C\ 35/45$

3.1.2: Tab. 3.1: E_{cm} for $C\ 35/45$

$E_{cm} = 34000\ N/mm^2$

3.3: Prestressing Steel

Prestressing Steel: $Y\ 1770$

3.3.6 (3): E_p for wires

$E_p = 195000\ N/mm^2$

3.3.2, 3.3.3: f_{pk} Characteristic tensile strength of prestressing steel

$f_{pk} = 1770\ N/mm^2$

3.3.2, 3.3.3: $f_{p0,1k}$ 0.1% proof-stress of prestressing steel, yield strength

$f_{p0,1k} = 1520\ N/mm^2$

Prestressing system: $BBV\ L19\ 150\ mm^2$

19 wires with area of $150\ mm^2$ each, giving a total of $A_p = 28.5\ cm^2$

Cross-section:

$A_c = 1.0 \cdot 1.0 = 1\ m^2$

Diameter of duct $\phi_{duct} = 97\ mm$

Ratio $\alpha_{E,p} = E_p\ /\ E_{cm} = 195000\ /\ 34000 = 5.74$

$A_{c,netto} = A_c - \pi \cdot (\phi_{duct}/2)^2 = 0.9926\ m^2$

$A_{ideal} = A_c + A_p \cdot \alpha_{E,p} = 1.013\ m^2$

5.10.2.1 (1)P: Prestressing force during tensioning - Maximum stressing force

5.10.2.1 (1)P: Eq. 5.41: P_{max} maximum stressing force

(NDP) 5.10.2.1 (1)P: $\sigma_{p,max}$ maximum stress applied to the tendon

The force applied to a tendon, i.e. the force at the active end during tensioning, should not exceed the following value

$P_{max} = A_p \cdot \sigma_{p,max}$

where $\sigma_{p,max} = \min\left\{0.80 f_{pk}; 0.90 f_{p0,1k}\right\}$

$P_{max} = A_p \cdot 0.80 \cdot f_{pk} = 28.5 \cdot 10^{-4} \cdot 0.80 \cdot 1770 = 4035.6\ kN$

$P_{max} = A_p \cdot 0.90 \cdot f_{p0,1k} = 28.5 \cdot 10^{-4} \cdot 0.90 \cdot 1520 = 3898.8\ kN$

$\rightarrow P_{max} = 3898.8\ kN$ and $\sigma_{p,max} = 1368\ N/mm^2$

5.10.3 (2): Prestress force

The value of the initial prestress force at time $t = t_0$ applied to the concrete immediately after tensioning and anchoring should not exceed the following value

5.10.3 (2): Eq. 5.43: P_{m0} initial prestress force at time $t = t_0$

$P_{m0}(x) = A_p \cdot \sigma_{p,m0}(x)$

[1]The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.3.6: Fig. 3.10, which can be seen in Fig 3.

[2]The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

where $\sigma_{p,m0}(x) = \min\{0.75 f_{pk}; 0.85 f_{p0,1k}\}$

(NDP) 5.10.3 (2): $\sigma_{p,m0}(x)$ stress in the tendon immediately after tensioning or transfer

$P_{m0} = A_p \cdot 0.75 \cdot f_{pk} = 28.5 \cdot 10^{-4} \cdot 0.75 \cdot 1770 = 3783.4\ kN$

$P_{m0} = A_p \cdot 0.85 \cdot f_{p0,1k} = 28.5 \cdot 10^{-4} \cdot 0.85 \cdot 1520 = 3682.2\ kN$

$\rightarrow P_{m0} = 3682.2\ kN$ and $\sigma_{p,m0} = 1292\ N/mm^2$

Further calculations for the distribution of prestress forces and stresses along the beam are not in the scope of this Benchmark and will not be described here. The complete diagram can be seen in Fig. 5, after the consideration of losses at anchorage and due to friction, as calculated by SOFiSTiK. There the values of $\sigma_{p,max} = 1368\ N/mm^2$ and $P_{m0} = 3682.2\ kN$ can be visualised.

Load Actions:

Self weight per length: $\gamma = 25\ kN/m$

$\rightarrow g_1 = \gamma \cdot A = 25 \cdot 1 = 25\ kNm$

Safety factors at ultimate limit state

DIN EN 1990/NA [4]: (NDP) A.1.3.1 (4): Tab. NA.A.1.2 (B): Partial factors for actions

(NDP) 2.4.2.2 (1): Partial factors for prestress

Actions (unfavourable)	Safety factor at final state
• permanent	$\gamma_G = 1.35$
• prestress	$\gamma_P = 1.00$

Combination coefficients at serviceability limit state

$g_1 = 25\ kNm$: for rare, frequent and quasi-permanent combination (for stresses)

At $x = 10.0\ m$ middle of the span:

$M_g = g_1 \cdot L^2\ /\ 8 = 1250\ kNm$

$N_p = P_{m0}(x = 10.0\ m) = -3653.0\ kN$ (from SOFiSTiK)

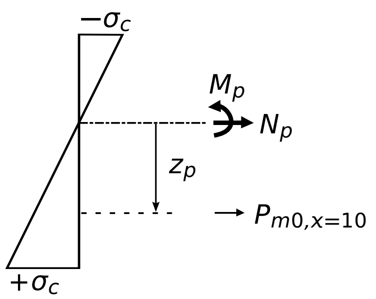
Calculation of stresses $\sigma_{c,b}$ at $x = 10.0\ m$ middle of the span:

The concrete stresses may be determined for each construction stage under the total quasi-permanent combination $\sigma_c\ \{G + P_{m0} + \psi_2 \cdot Q\}$

In this Benchmark no variable load Q is examined

Position of the tendon: $z = 0,3901\ m$

- *Case I*: Prestress at construction stage section 0 (P cs0)



$N_p = -3653.0\ kN$

z_s the new position of the center of the cross-section for cs0

$z_p = z + z_s$

M_p bending moment caused by prestressing

$W_{1,cs0}$ cross-section moduli for contruction stage 0 at the bottom left and right point

$\sigma_{c,b}$ stress at the concrete at the bottom of the cross section

$$M_{p_1} = N_P \cdot z = -3653.0 \cdot 0.3901 = -1425.04\ kNm$$

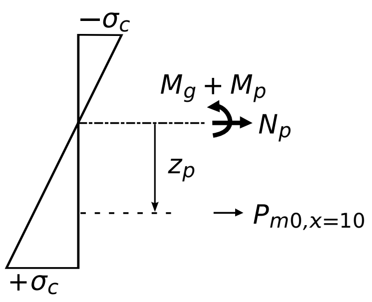
$$M_{p_2} = N_P \cdot z_s = -3653.0 \cdot 0.002978 = -10.879\ kNm$$

$$M_p = -1425.04 - 10.879 = -1435.91\ kNm = M_y$$

$$\sigma_{c,b} = \frac{N_p}{A_{c,netto}} + \frac{M_y}{W_{1,cs0}}$$

$$\sigma_{c,b} = \frac{-3653.0}{0.9926} + \frac{-1435.91}{0.1633} = -12.47\ MPa$$

- *Case II*: Prestress and self-weight at con. stage sect. 0 (P+G cs0)



$N_p = -3653.0\ kN$ and $M_g = 1250\ kNm$

As computed above: $M_p = -1435.91\ kNm$

$$M_y = 1250 - 1435.91 = -185.91\ kNm$$

$$\sigma_{c,b} = \frac{-3653.0}{0.9926} + \frac{-185.91}{0.1633} = -4.82\ MPa$$

- *Case III*: Prestress at con. stage sect. 1 (P cs1)

$N_p = -3653.0\ kN$ and $M_{p_1} = -1425.04\ kNm$ (as above)

$$M_{p_2} = N_P \cdot z_s = -3653.0 \cdot (-0.00518) = 18.92\ kNm$$

$$M_p = -1425.04 + 18.92 = -1406.11\ kNm = M_y$$

$W_{1,cs1}$ cross-section moduli for contruction stage 1

$$\sigma_{c,b} = \frac{N_p}{A_{ideal}} + \frac{M_y}{W_{1,cs1}}$$

$$\sigma_{c,b} = \frac{-3653.0}{1.013} + \frac{-1406.11}{0.172} = -11.76\ MPa$$

- *Case IV*: Prestress and self-weight at con. stage sect. 1 (P+G cs1)

$N_p = -3653.0\ kN$ and $M_g = 1250\ kNm$

As computed above: $M_p = -1406.11\ kNm$

$$M_y = 1250 - 1406.11 = -156.11\ kNm$$

$$\sigma_{c,b} = \frac{-3653.0}{1.013} + \frac{-156.11}{0.172} = -4.51\ MPa$$

5 Conclusion

This example shows the calculation of the stresses, developed in the concrete cross-section due to prestress and bending. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.

[3] *Beispiele zur Bemessung nach Eurocode 2 - Band 1: Hochbau*. Ernst & Sohn. Deutschen Beton- und Bautechnik-Verein E.V. 2011.

[4] *DIN EN 1990/NA: Eurocode: Basis of structural design, Nationaler Anhang Deutschland DIN EN 1990/NA:2010-12*. CEN. 2010.