Benchmark Example No. 11

Shear at the interface between concrete cast

SOFiSTiK | 2024

VERiFiCATiON
DCE-EN11 Shear at the interface between concrete cast

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SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

6th Street Viaduct, Los Angeles   Photo: Tobias Petschke

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB
Input file(s):	shear_interface.dat

1 Problem Description

The problem consists of a T-beam section, as shown in Fig. 1. The cs is designed for shear, the shear at the interface between concrete cast at different times is considered and the required reinforcement is determined.

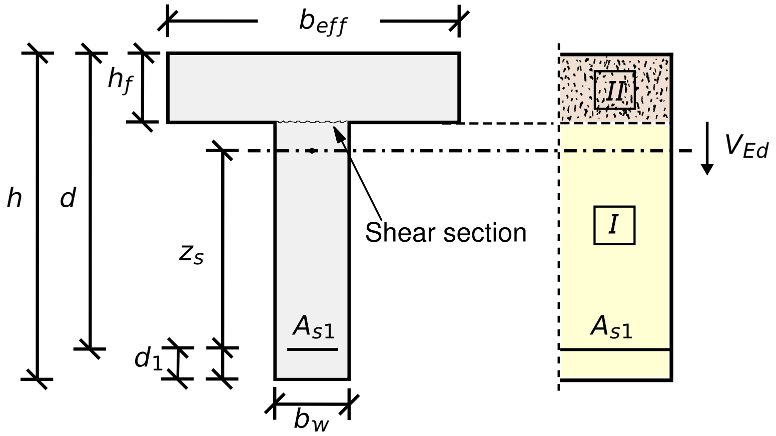

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the shear design of T-sections, for the ultimate limit state. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.3)
- Guidelines for shear design (Section 6.2)

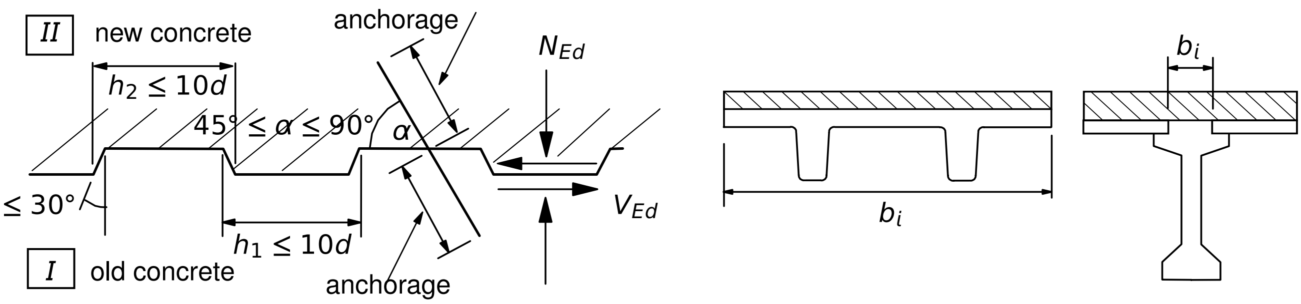

Figure 2: Indented Construction Joint - Examples of Interfaces

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).

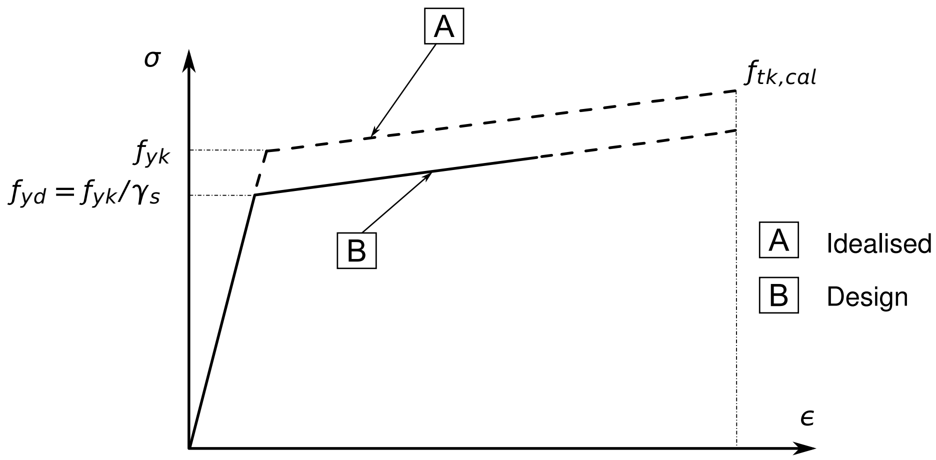

Figure 3: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The T-section, with properties as defined in Table 1, is to be designed for shear, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2]. The reference calculation steps [3] are presented in the next section and the results are given in Table 2.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
C 20/25	$h = 135.0\ cm$	$V_z = 800\ kN$
B 500A	$h_f = 29 cm$	$M_y = 2250\ kNm$
	$d_1 = 7.0\ cm$	
	$b_w = 40\ cm$, $b_{eff} = 250\ cm$	
	$A_{s1} = 1.0\ cm^2$	
	$z_s = 95.56\ cm$	

Table 2: Results

$a_s\ [cm^2/m]$	SOF.	Ref.
state I	7.00	7.07
state II only V	4.86	4.90
state II $V + M$	4.99	4.99

4 Design Process [1]

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

Material:

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

$f_{ck} = 25\ MPa$

$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 0.85 \cdot 25/1.5 = 14.17\ MPa$

Tab. 3.1: Strength for concrete

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$f_{yk} = 500\ MPa$

$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ MPa$

3.2.2: (3)P: yield strength $f_{yk} = 500\ MPa$

3.2.7: (2), Fig. 3.8

$\sigma_{sd} = 456.52\ MPa$

$$\tau = \frac{T_v}{b_w} = \frac{V \cdot S}{I_y \cdot b_w}$$

where S is the static moment of the separated area

$S = h_w \cdot b_w \cdot (z_s - h_w/2) = 0.18058\ m^3$

$$\tau = \frac{0.8 \cdot 0.18058}{0.16667 \cdot 0.4} = 2.1669\ MPa$$

$$T_v = \frac{0.8 \cdot 0.18058}{0.16667} = 0.86676\ MN/m = 866.76\ kN/m$$

The shear section with a length of 0.4 m is split into two equal parts with $b_i = 0.2\ m$

$T_v = 866.76\ /\ 2 = 433.38\ kN/m$

State I:

Design Load:

The associated design shear flow V_{Edi} is:

$V_{Edi} = T_v = 433.38\ kN/m$

$v_{Edi} = \tau = 2.1669\ MPa$

$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + 0.12 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$

(NDP) 6.2.2 (1): Eq. 6.2a: Design value for shear resistance $V_{Rd,c}$ for members not requiring design shear reinforcement

$v_{Rd,c} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + 0.12 \cdot \sigma_{cp}$

$\rho_1 = \dfrac{A_{sl}}{b_w d} = 0.0 \ \rightarrow v_{Rd,c} = 0.0$

with a minimum of

$V_{Rd,c,min} = (\nu_{min} + 0.12 \cdot \sigma_{cp}) \cdot b_w \cdot d$

(NDP) 6.2.2 (1): Eq. 6.2b

$v_{Rd,c,min} = \nu_{min} + 0.12 \cdot \sigma_{cp}$

$\nu_{min} = (0.0375/\gamma_c) \cdot k^{3/2} \cdot f_{ck}^{\ 1/2} = 0.20833\ MPa$

(NDP) 6.2.2 (1): Eq. 6.3bDE

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

[2] The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

$v_{Rd,c,min} = 0.20833 \rightarrow v_{Rd,c} = 0.20833\ MPa$

$v_{Edi} > v_{Rd,c} \rightarrow$ shear reinforcement is required

6.2.5 (1): Eq. 6.23: The design shear stress at the interface should satisfy this

$v_{Edi} \leq v_{Rdi}$

(NDP) 6.2.5 (1): Eq. 6.25

$v_{Rdi} = c \cdot f_{ctd} + \mu \cdot \sigma_n + \rho \cdot f_{yd} \cdot (1.2 \cdot \mu \cdot \sin\alpha + \cos\alpha)$

Maximum shear stress $v_{Rdi,max}$

(NDP) 6.2.5 (1): $\nu = 0.70$ for indented surface

and $v_{Rdi} \leq 0.5 \cdot \nu \cdot f_{cd}$

$v_{Rdi,max} = 0.5 \cdot \nu \cdot f_{cd} = 4.9585\ MPa$

6.2.5 (2): c, μ: factors depending on the roughness of the interface

$c = 0.50$ and $\mu = 0.9$ for indented surface

(NDP) 3.1.6 (2)P: Eq. 3.16

$\alpha_{ct} = 0.85$

3.1.2 (3): Tab. 3.1 - Strength for concrete: $f_{ctk;0.05} = 1.8\ MPa$:

$f_{ctd} = \alpha_{ct} \cdot f_{ctk;0.05} \ / \ \gamma_c$

$f_{ctd} = 0.85 \cdot 1.80 \ / \ 1.5 = 1.02$

$\rho = \frac{a_s}{b_i \cdot l_i}$: area of reinforcement crossing interface / area of joint

$$v_{Rdi} = 0.5 \cdot 1.02 + 0 + \frac{a_s}{0.2 \cdot 1.0} \cdot 435 \cdot (1.2 \cdot 0.9 \cdot 1 + 0)$$

$$v_{Rdi} = 0.51 + \frac{a_s}{0.2} \cdot 469.56 = 2.1669$$

$a_s = 7.07\ cm^2/m$

State II only shear force V:

Design Load:

From the calculated inner lever arms for the two states we get a ratio:

$$\frac{z_I}{z_{II}} = 0.7664$$

The associated design shear flow V_{Edi} is:

$V_{Edi} = 0.7664 \cdot 433.38 = 332.15\ kN/m$

and $v_{Edi} = 332.15/0.2 = 1.66\ MPa$

Following the same calculation steps as for state II we have:

$v_{Rd,c} = 0.20833\ MPa$ (as above)

$v_{Edi} > v_{Rd,c} \rightarrow$ shear reinforcement is required

$v_{Edi} \leq v_{Rdi}$

$v_{Rdi} = c \cdot f_{ctd} + \mu \cdot \sigma_n + \rho \cdot f_{yd} \cdot (1.2 \cdot \mu \cdot \sin\alpha + \cos\alpha)$

$$v_{Rdi} = 0.5 \cdot 1.02 + 0 + \frac{a_s}{0.2 \cdot 1.0} \cdot 435 \cdot (1.2 \cdot 0.9 \cdot 1 + 0)$$

$$v_{Rdi} = 0.51 + \frac{a_s}{0.2} \cdot 469.56 = 1.66$$

$a_s = 4.90\ cm^2/m$

State II shear force V and moment M:

$M_{Eds} = 2250\ kNm$

$$\mu_{Eds} = \frac{M_{Eds}}{b_{eff} \cdot d^2 \cdot f_{cd}} = \frac{2250 \cdot 10^{-3}}{2.5 \cdot 1.28^2 \cdot 14.17} = 0.03876$$

$\omega = 0.03971$ and $\xi = 0.9766$ (interpolated)

Tab. 9.2 [4]: ω—Table for up to $C50/60$ without compression reinforcement

$$A_{s1} = \frac{1}{\sigma_{sd}} \cdot (\omega \cdot b \cdot d \cdot f_{cd} + N_{Ed}) = 39.44\ cm^2$$

$N_{Ed} = 0$

$z = \max\left\{d - c_{V,l} - 30\ mm;\ d - 2\ c_{V,l}\right\}$

(NDP) 6.2.3 (1): Inner lever arm z

$z = \max\{1160;\ 1190\} = 1190\ mm$

Design Load:

$T_v = V\ /\ z = 800\ /\ 1.19 = 672.268\ kN/m$

The shear section with a length of 0.4 m is split into two equal parts with $b_i = 0.2\ m$

$T_v = 672.268\ /\ 2 = 336.134\ kN/m$

$V_{Edi} = 336.134\ kN/m$

and $v_{Edi} = 336.134\ /\ 0.2 = 1.68\ MPa$

$v_{Rd,c} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + 0.12 \cdot \sigma_{cp}$

$C_{Rd,c} = 0.15/\gamma_c = 0.1$

(NDP) 6.2.2 (1): $C_{Rd,c} = 0.15/\gamma_c$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{1280}} = 1.3953 < 2.0$$

$$\rho_1 = \frac{A_{sl}}{b_w d} = \frac{39.44}{40 \cdot 128} = 0.007703 < 0.02$$

$v_{Rd,c} = 0.1 \cdot 1.3953 \cdot (100 \cdot 0.007703 \cdot 25)^{1/3} + 0$

$v_{Rd,c} = 0.373229\ MPa$

$v_{Edi} > v_{Rd,c}\ \rightarrow$ shear reinforcement is required

$v_{Rdi} = c \cdot f_{ctd} + \mu \cdot \sigma_n + \rho \cdot f_{yd} \cdot (1.2 \cdot \mu \cdot \sin\alpha + \cos\alpha)$

$$v_{Rdi} = 0.5 \cdot 1.02 + 0 + \frac{a_s}{0.2 \cdot 1.0} \cdot 435 \cdot (1.2 \cdot 0.9 \cdot 1 + 0)$$

$$v_{Rdi} = 0.51 + \frac{a_s}{0.2} \cdot 469.56 = 1.68$$

$a_s = 4.99\ cm^2/m$

5 Conclusion

This example shows the calculation of the required reinforcement for a T-section under shear at the interface between concrete cast at different times. It has been shown that the results are reproduced with excellent accuracy. Small deviations occur because AQUA calculates (by using FEM analysis) the shear stresses more accurate compared to the reference example.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.

[3] *Beispiele zur Bemessung nach Eurocode 2 - Band 1: Hochbau*. Ernst & Sohn. Deutschen Beton- und Bautechnik-Verein E.V. 2011.

[4] K. Holschemacher, T. Müller, and F. Lobisch. *Bemessungshilfsmittel für Betonbauteile nach Eurocode 2 Bauingenieure*. 3rd. Ernst & Sohn, 2012.