Benchmark Example No. 8

Large Deflection of Cantilever Beams II

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VERiFiCATiON
BE8 Large Deflection of Cantilever Beams II

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Front Cover

6th Street Viaduct, Los Angeles   Photo: Tobias Petschke

Overview	
Element Type(s):	B3D, SH3D
Analysis Type(s):	STAT, GNL
Procedure(s):	LSTP
Topic(s):	
Module(s):	ASE
Input file(s):	moment_beam.dat, moment_quad.dat

1 Problem Description

The cantilever beam of Benchmark Example No. 7 is analysed here for a moment load, as shown in Fig. 1, with both beam and quad plane elements. The accuracy of the elements is evaluated through the deformed shape of the beam retrieved by limit load iteration procedure.

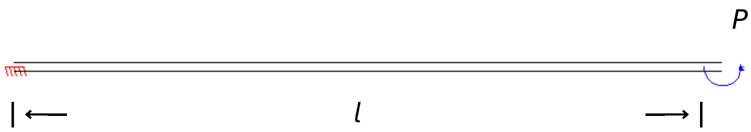


Figure 1: Model Properties

2 Reference Solution

The classical problem of deflection of a cantilever beam of linear elastic material, is here extended for the case of a moment applied at the beam tip. The concentrated moment causes the beam to wind around itself, i.e. deflect upwards and bend towards the built-in end. The analytical solution can be derived from the fundamental Bernoulli-Euler theory, which states that the curvature of the beam at any point is proportional to the bending moment at that point [1]. For the case of pure bending, the beam will bend into a circular arc of curvature R

$$R = \frac{EI}{M}, \tag{1}$$

and will wind n times around itself [2]

$$\frac{ML}{EI} = 2\pi n, \tag{2}$$

where I is the moment of inertia, E the Elasticity modulus and M the concentrated moment applied at the tip.

3 Model and Results

The properties of the two models analysed are defined in Table 1. For the moment load, the deformed shape of the structure for quad elements at various increments throughout the steps, are shown in Fig. 2. According to the analytical solution and the moment load applied, the cantilever is expected to wind around itself $n = 2$.

Table 1: Model Properties

Material Properties	Geometric Properties Beam elements	Geometric Properties Quad elements	Loading
$E = 100\,MPa$	$l = 10\,m$	$l = 10\,m$	$M = 3.38478\,kNm$
	$D = 0.2\,m$	$B = 0.3\,m$	
	$t = 0.01\,m$	$t = 0.10261\,m$	

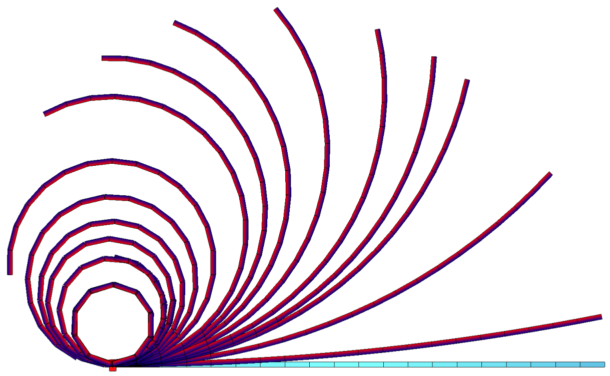


Figure 2: Deformed Structure - Quad Elements

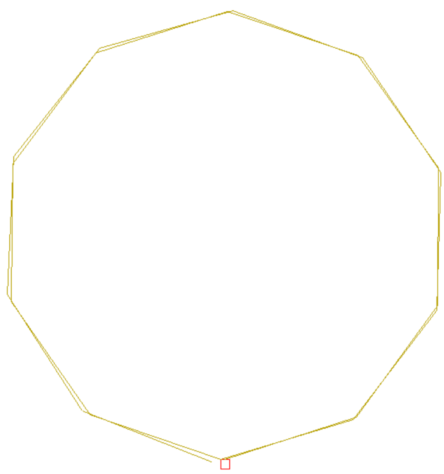
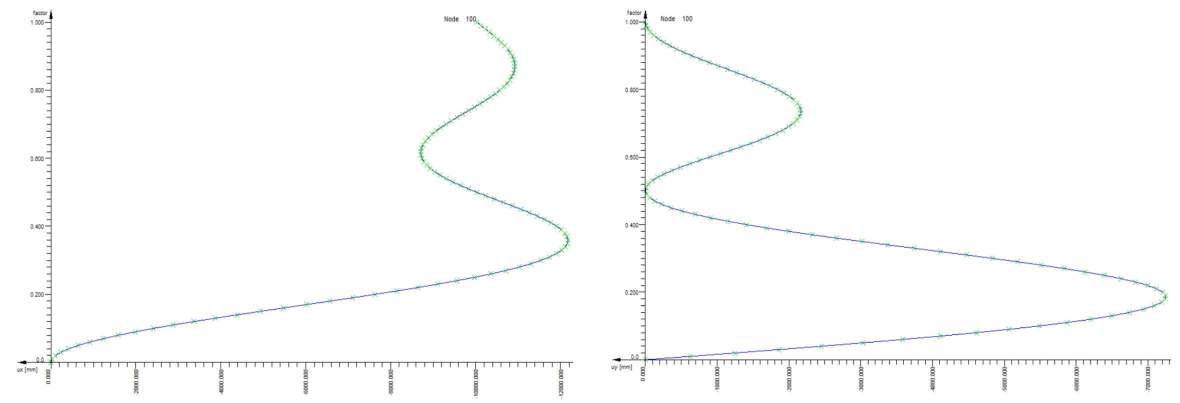
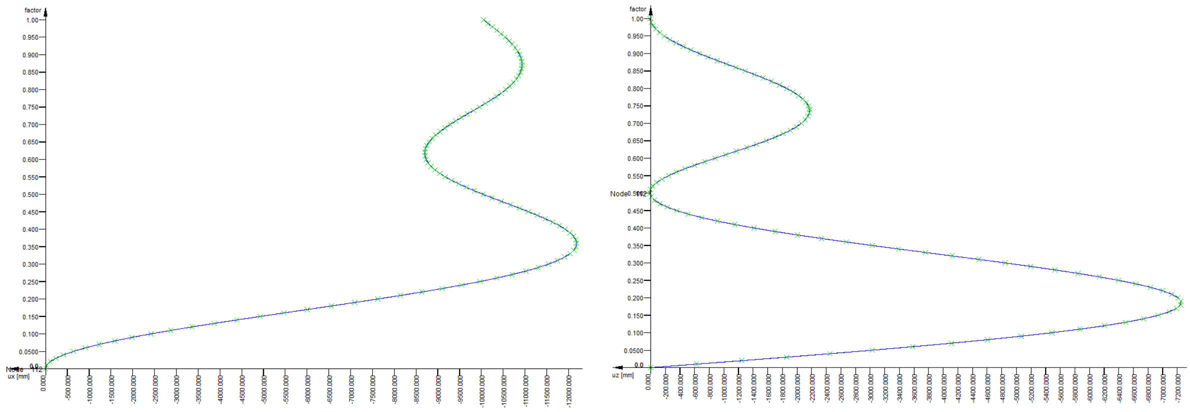


Figure 3: Final Deformed Shape of Cantilever with Quad Elements

Figure 4 presents the load - deflection curve for the horizontal and vertical direction for the two cases. From the final deformed shape of the beam (Fig. 3), it is evident that the cantilever achieves $n \approx 2$, which can also be observed at the second load-deflection curve where the vertical displacement becomes zero twice.



Beam Elements



Quad Elements

Figure 4: Load - Deflection Curve

4 Conclusion

This benchmark shows the classical problem of a cantilever beam undergoing large deformations under the action of a moment load applied at the tip. The accuracy of the deformation solution for the quad and beam elements is evident.

5 Literature

[1] A. A. Becker. *Background to Finite Element Analysis of Geometric Non-linearity Benchmarks*. Tech. rep. NAFEMS, 1998.

[2] *Abaqus Benchmarks Manual 6.10*. Dassault Systémes Simulia Corp. 2010.