Benchmark Example No. 29

Cylindrical Hole in an Infinite Mohr-Coulomb Medium

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VERiFiCATiON
BE29 Cylindrical Hole in an Infinite Mohr-Coulomb Medium

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Front Cover

6th Street Viaduct, Los Angeles Photo: Tobias Petschke

Overview	
Element Type(s):	C2D
Analysis Type(s):	STAT, MNL
Procedure(s):	LSTP
Topic(s):	SOIL
Module(s):	TALPA
Input file(s):	hole_mohr.dat

1 Problem Description

This problem verifies stresses for the case of a cylindrical hole in an infinite elastic-plastic medium subjected to a constant in-situ state, as shown in Fig. 1. The material is assumed to be linearly elastic and perfectly plastic with a failure surface defined by the Mohr-Coulomb criterion. The stresses and the displacements are verified.

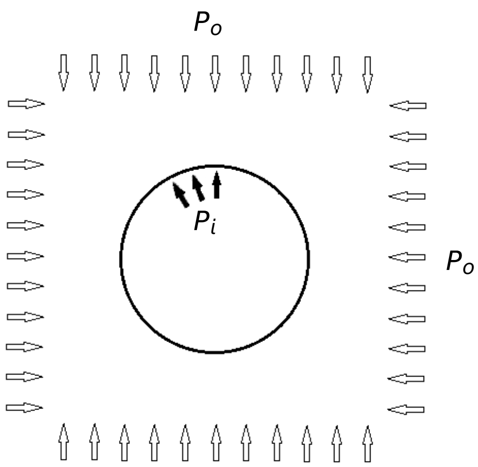

Figure 1: Problem Description

2 Reference Solution

Consider a hollow cylinder with inner radius α and outer radius r, under plane strain conditions, with a uniform pressure applied to its outer surface. If this pressure is slowly increased from 0 to some value P_o, at first the cylinder will everywhere be in the elastic zone. As Po increases further, the yielding will start, the yielded zone will grow radially outward, and the cylinder will consist of an inner annular region that has yielded and an outer annulus that is still in its elastic state [1]. A specialised problem is now the calculation of the stresses outside a cylindrical hole in an infinite elastic-perfectly-plastic medium, here with a failure surface defined by the Mohr-Coulomb criterion. Assume that the rock mass is initially under hydrostatic stress P_o and then a circular hole of radius α is drilled into the rock, so that the stress at $r = \alpha$ is reduced to some value P_i. The yield zone radius R_o is given analytically by the theoretical model based on the solution of Salencon [2]

$$R_o = \alpha \left(\frac{2}{K_p + 1} \frac{P_o + \frac{q}{K_p - 1}}{P_i + \frac{q}{K_p - 1}} \right)^{\frac{1}{K_p - 1}} \tag{1}$$

where α is the radius of the hole, P_o the initial in-situ stress, P_i the internal pressure and K_p, q are given by

$$K_p = \frac{1 + sin\,\phi}{1 - sin\,\phi} \tag{2}$$

$$q = 2c\,tan\,(45 + \phi/2) \tag{3}$$

The parameters c and ϕ correspond to the cohesion and angle of friction of the medium respectively. For sufficiently small values of P_o, where P_o ¡ P_i holds, the medium will be in its elastic state, and the stresses will be given by [1] [3]

$$\sigma_r = P_o - (P_o - \sigma_{re})\left(\frac{R_o}{r}\right)^2 \tag{4}$$

$$\sigma_\theta = P_o + (P_o - \sigma_{re})\left(\frac{R_o}{r}\right)^2 \tag{5}$$

where r is the distance from the field point to the center of the hole and σ_{re} is the radial stress at the elastic-plastic interface

$$\sigma_{re} = \frac{1}{K_p + 1}(2P_o - q) \tag{6}$$

For $P_o > P_i$, the rock will fail within some annular region surrounding the borehole. The stresses in the yielded zone will be given by

$$\sigma_r = -\frac{q}{K_p - 1} + \left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{r}{\alpha}\right)^{K_p - 1} \tag{7}$$

$$\sigma_r = -\frac{q}{K_p - 1} + K_p\left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{r}{\alpha}\right)^{K_p - 1} \tag{8}$$

3 Model and Results

The properties of the model are defined in Table 1. The radius of the hole is 1 m and is assumed to be small compared to the length of the cylinder, therefore $2D$ plane strain conditions are in effect. A fixed external boundary is located 29.7 m from the hole center. The model is presented in Fig. 2. The stresses are calculated and verified with respect to the formulas provided in Section 2.

Table 1: Model Properties

Material Properties	Geometric Properties	Pressure Properties
$E = 6777.9\ MPa$	$\alpha = 1\ m$	$P_o = 30\ MPa$
$\nu = 0.21$	$r_{boundary} = 29.7\ m$	$P_i = 0$ or $1\ MPa$

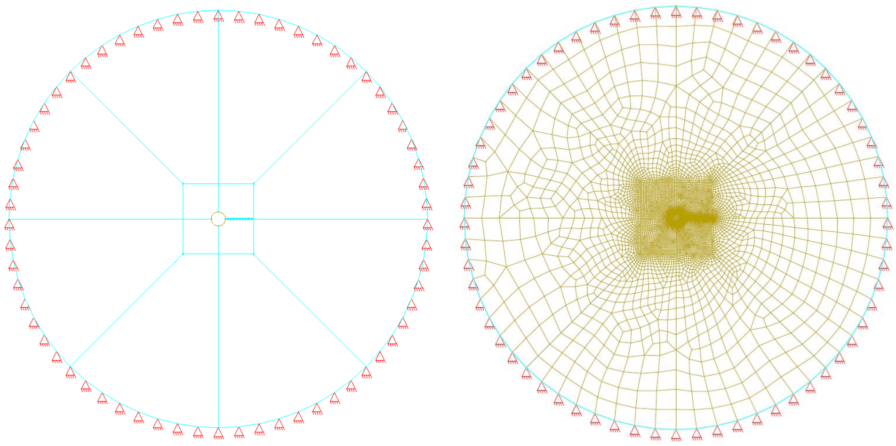

Figure 2: Finite Element Model

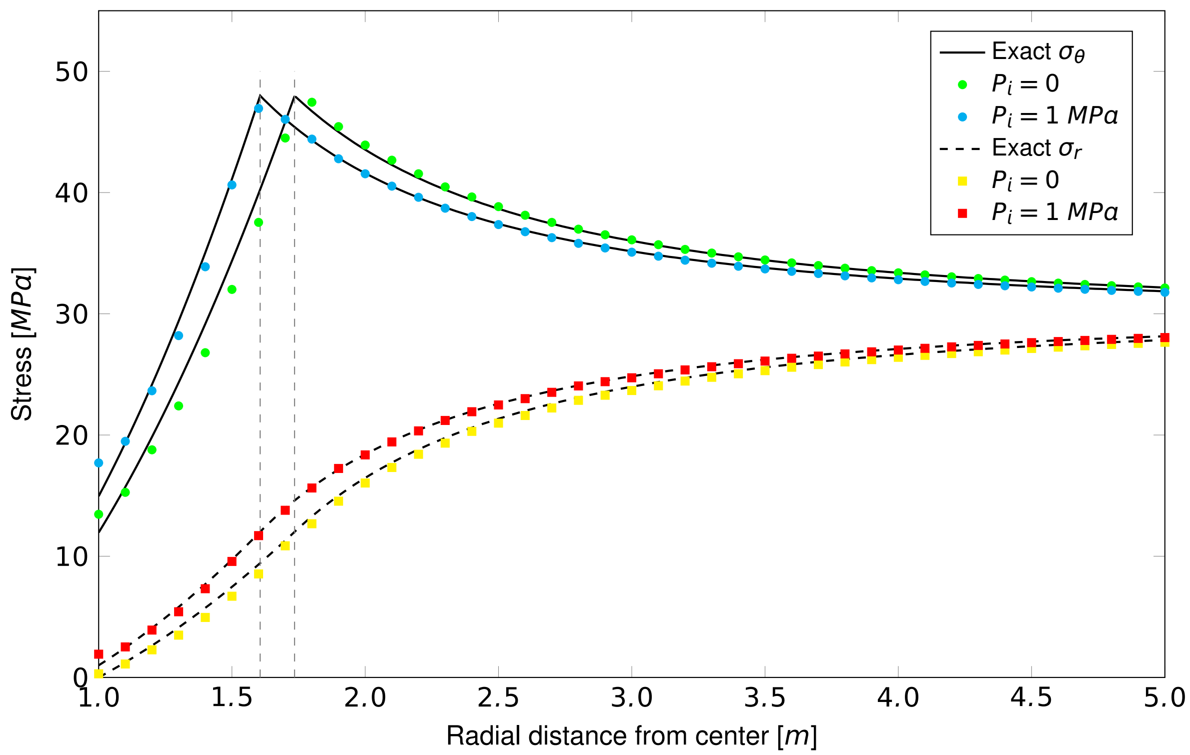

Figure 3: Radial and Tangential Stresses for Cylindrical Hole in Infinite Mohr-Coulomb Medium

Figure 3 show the radial and tangential stress, along a line, lying on the X-axis. Results are presented for two cases, first with no internal pressure and second with $P_i = 1\ MPa$. The results in both cases are in very good agreement with the reference solution.

4 Conclusion

This example verifies the stresses of a cylindrical hole in an infinite elastic-perfectly-plastic medium. It has been shown that the behaviour of the model is captured accurately.

5 Literature

[1] J. C. Jaeger, N. G. W. Cook, and R. W. Zimmerman. *Fundamentals of Rock Mechanics*. 4th. Blackwell Publishing, 2007.
[2] J. Salencon. “Contraction Quasi-Statique D’ une Cavite a Symetrie Spherique Ou Cylindrique Dans Un Milieu Elasto-Plastique”. In: *Annales Des Ports Et Chaussees* 4 (1969).
[3] *Phase 2 Stress Analysis Verification Manual Part I*. Rocscience Inc. 2009.