SOFiSTiK

Benchmark Example No. 27

Response of a SDOF System to Impulsive Loading

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VERiFiCATiON
BE27 Response of a SDOF System to Impulsive Loading

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SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

6th Street Viaduct, Los Angeles   Photo: Tobias Petschke

Overview	
Element Type(s):	SPRI
Analysis Type(s):	DYN
Procedure(s):	TSTP
Topic(s):	
Module(s):	DYNA
Input file(s):	impulse_sine_wave.dat, impulse_rectangular.dat

1 Problem Description

This problem consists of an elastic undamped SDOF system undergoing forced vibration (Fig. 1) due to an impulsive loading as the one shown in Fig. 2. The response of the system is determined and compared to the exact reference solution.

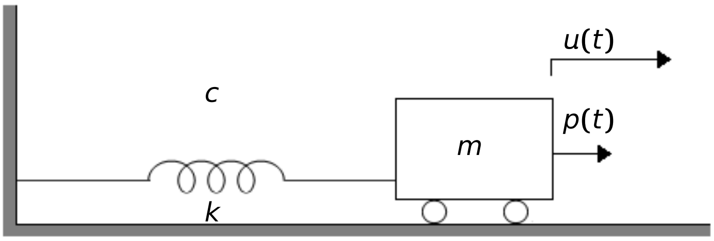


Figure 1: Problem Description

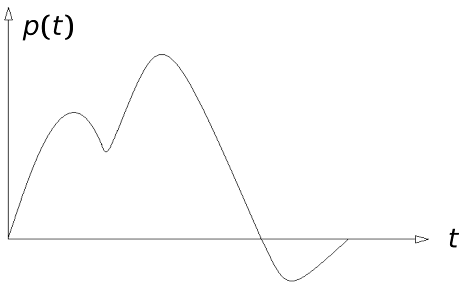
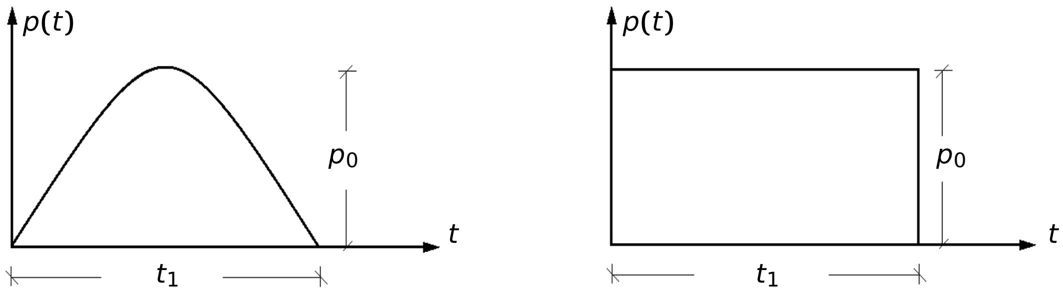


Figure 2: Arbitrary Impulsive Loading

2 Reference Solution

Another special case of dynamic loading of the SDOF system is the impulsive load. Such a load consists of a single principal impulse of arbitrary form, as illustrated in Fig. 2, and generally is of relatively short duration. Damping has much less importance in controlling the maximum response of a structure to impulsive loads than for periodic or harmonic loads because the maximum response to a particular impulsive load will be reached in a very short time, before the damping forces can absorb much energy from the structure [1]. Therefore the undamped response to impulsive loads will be considered in this Benchmark.



(a) Half sine wave

(b) Rectangular

Figure 3: Examined Impulse Loading

The response to an impulse loading is always divided into two phases, the first corresponds to the forced vibration phase in the interval during which the load acts and the second corresponds to the free vibration phase which follows. Let us consider the case, where the structure is subjected to a single half sine wave loading as shown in Fig. 3(a). Assuming that the system starts from rest, the undamped response ratio time history $R(t) = u(t)/(p_0/k)$, is given by the simple harmonic load expression

$$R(t) = \frac{1}{1-\beta^2}\left[sin\ \omega_p t - \beta\ sin\ \omega_n t\right] \tag{1}$$

where $\beta = \omega_p/\omega_n$, p_o is the amplitude value of the force and ω_p its frequency. Introducing the non dimensional time parameter $\alpha = t/t_1$ so that $\omega_p t = \pi\alpha$ and $\omega_n t = \pi\alpha/\beta$, we can rewrite the equation accordingly

$$R(\alpha) = \frac{1}{1-\beta^2}\left[sin\ \pi\alpha - \beta\ sin\ \frac{\pi\alpha}{\beta}\right] \qquad 0 \leq \alpha \leq 1 \tag{2}$$

where t_1 the duration of the impulse and $\beta \equiv T/2t_1$. This equation is valid only for phase I corresponding to $0 \leq \alpha \leq 1$. While it is very important to understand the complete time history behaviour as shown in Fig. 4, the engineer is usually only interested in the maximum value of response as represented by Points a, b, c, d, and e. If a maximum value occurs in Phase I, the value of α at which it occurs can be determined by differentiating Eq. 2 with respect to α and equating to zero

$$\frac{d\,R(\alpha)}{d\,\alpha} = 0 \tag{3}$$

solving for α yields the α values for the maxima

$$\alpha = \frac{2\beta n}{\beta + 1} \qquad n = 0, 1, 2, \ldots \qquad 0 \leq \alpha \leq 1 \tag{4}$$

For phase II where $t \geq t_1$ and the free vibration occurs, the value of α is not necessary and the maximum response is given by

$$R_{max} = \left[\frac{-2\beta}{1-\beta^2}\right] cos\,\frac{\pi}{2\beta} \qquad \alpha \geq 1 \tag{5}$$

Accordingly for the case of a rectangular impulse loading Fig. 3(b), the general response ratio solution for at rest initial conditions and for phase I is given by

$$R\,(\alpha) = 1 - cos\,2\pi\,\frac{t_1}{T}\,\alpha \qquad 0 \leq \alpha \leq 1 \tag{6}$$

The maximum response ratio R_{max} is given again in terms of α and can be determined in the same manner by differentiating Eq. 6 with respect to α and equating to zero, yielding

$$\alpha = \beta n \qquad n = 0, 1, 2, \ldots \qquad 0 \leq \alpha \leq 1 \tag{7}$$

For phase II, the maximum response of the free vibrating system is given by

$$R_{max} = 2\,sin\,\pi\,\frac{t_1}{T} \qquad \alpha \geq 1 \tag{8}$$

Special attention has to be given in the case of β = 1 where the expression of the response ratio becomes indeterminate and the L' Hospital's rule has to be utilised.

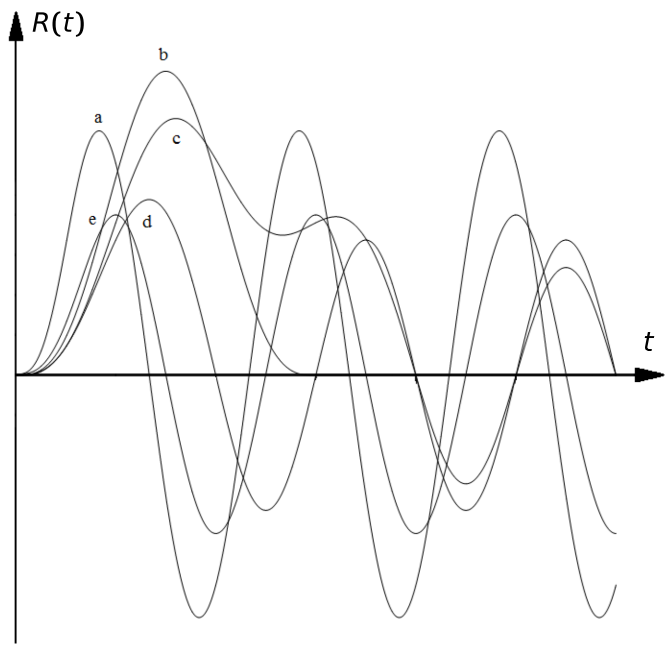


Figure 4: Response Ratios due to Half Sine Pulse

3 Model and Results

In the expressions derived before, the maximum response produced in an undamped SDOF structure by each type of impulsive loading depends only on the ratio of the impulse duration to the natural period of the structure t_1/T. Thus, it is useful to plot the maximum value of response ratio $Rmax$ as a function of t_1/T for various forms of impulsive loading. Such plots are commonly known as displacement-response spectra and are derived here, for two forms of loading, a rectangular and a half sine wave impulse. Generally plots like these can be used to predict with adequate accuracy the maximum effect to be expected from a given type of impulsive loading acting on a simple structure. The properties of the model are defined in Table 1. The resulting figures are presented in Fig. 5.

Table 1: Model Properties

Model Properties	Excitation Properties
$m = 1\,t$	$u(0) = 0$
$k = 4\pi^2\ kN/m$	$u(\dot{0}) = 0$
$T = 1\,sec$	$p_0 = 10\,kN$

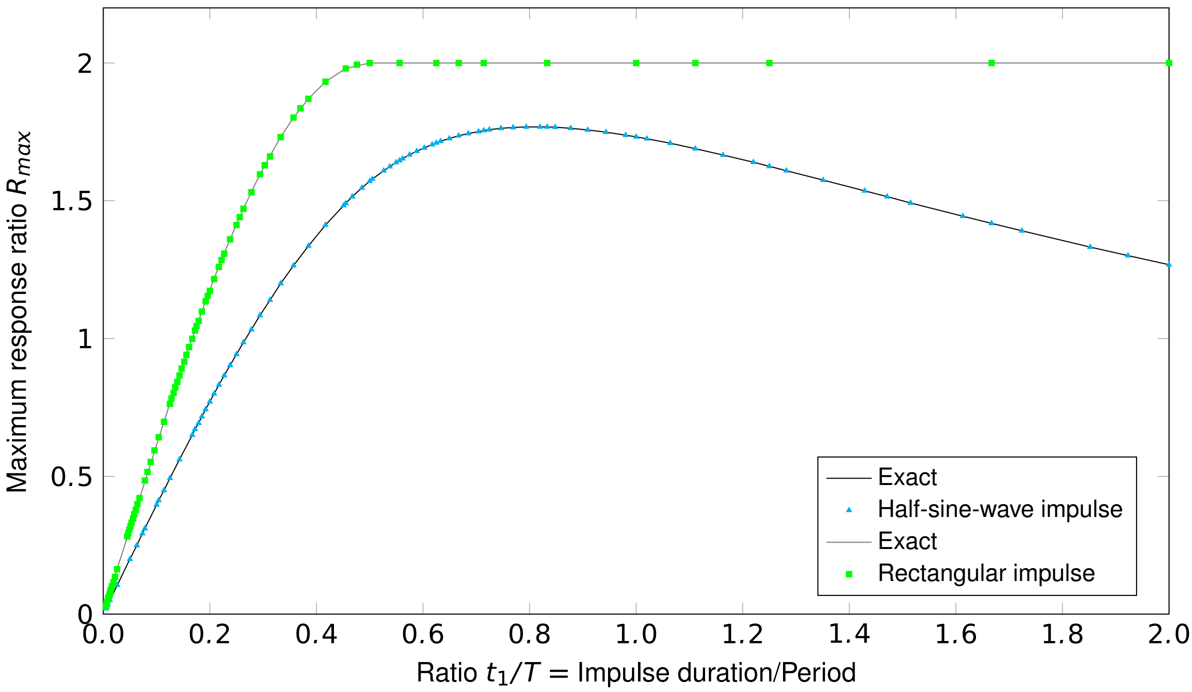


Figure 5: Displacement - Response Spectra for Two Types of Impulse

4 Conclusion

The purpose of this example is to test the calculation of the response of a dynamic system in terms of an impulsive loading. It has been shown that the behaviour of the system is captured adequately.

5 Literature

[1] R. W. Clough and J. Penzien. *Dynamics of Structures*. 3rd. Computers & Structures, Inc., 2003.